Dynamic Hedge Fund Portfolio Construction: A Semi-Parametric Approach

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Paper No: 11/05

October 2011

Abstract

In this paper, we evaluate alternative optimization frameworks for constructing portfolios of hedge funds. Using monthly hedge fund index returns for the period 1990 to 2011, we compare the standard mean-variance optimization model with models based on CVaR, CDaR and Omega, for both conservative and aggressive hedge fund investment strategies. In order to implement the CVaR, CDaR and Omega optimization models, we propose a semi-parametric methodology, in which we first model the marginal density of each hedge fund index using extreme value theory and construct the joint density of hedge fund index returns using a copula-based approach. We then simulate hedge fund returns from this joint density in order to compute CVaR, CDaR and Omega, which are used in the optimization process. We compare the semi-parametric approach with the standard, non-parametric approach, in which the quantiles of the marginal density of portfolio returns are estimated empirically and used to compute CVaR, CDaR and Omega. We report two main findings. The first is that the CVaR, CDaR and Omega models offer a significant improvement in terms of risk-adjusted portfolio performance over the mean-variance model. The second is that semi-parametric estimation of the CVaR, CDaR and Omega models offers a very substantial improvement over non-parametric estimation. Our results are robust to the choice of target return, risk limit and estimation sample size.

Keywords: Funds of hedge funds; Portfolio optimization; Copula; Extreme value theory; Monte Carlo simulation.

1. Introduction

Hedge funds have attracted much interest not only for their ability to generate relatively high average returns, but also for the large losses that they can incur, a risk that is exemplified by the rise and fall of Long Term Capital Management in the late 1990s. In spite of such risk, the hedge fund industry witnessed rapid growth in the 2000s, with assets under management reaching \$1.93 trillion by early 2008. During the recent credit crisis, there was a significant reduction both in the number of hedge funds and in assets under management, which resulted from a combination of trading losses and asset withdrawals by investors. However, by April 2011, it was estimated that hedge fund assets had recovered to their pre-crisis level (see Strasberg & Eder, 2011). An important contributing factor to this recent growth has been the availability of funds of hedge funds, which enable investors to access hedge fund alpha with lower risk, albeit at the expense of an additional layer of fees. Another contributing factor to the growth in the hedge fund industry was the launch in the early 2000s of investable hedge fund indices. These have generated further interest from small- and medium-sized investors, who would otherwise be precluded from investing in the hedge fund market. Central to both of these developments is the role of portfolio optimization in order to construct portfolios of individual hedge funds or investible hedge fund indices.

A number of studies have examined portfolio optimization in a hedge fund context. However, the optimal portfolio allocation across individual hedge funds is complicated by the fact that owing to the strategies that hedge fund managers typically adopt, hedge fund returns are far from normally distributed, often exhibiting very significant negative skewness and excess kurtosis (see, for example, Amin & Kat, 2001; Lo, 2001; Brooks & Kat, 2002; Fung & Hsieh, 1997a, 2001; Agarwal & Naik, 2004, Hudson *et al.*, 2006; Wegener *et al.*, 2010). Portfolio optimization in the presence of such non-normality generally leads to very different portfolio allocations than those implied by mean-variance analysis (see, for example, McFall Lamm, 2003; Fung & Hsieh, 1997b; Cvitanic *et al.*, 2003; Terhaar *et al.*, 2003; Popova *et al.*, 2003; Glaffig, 2006; Wong *et al.*, 2008). Motivated by the well established volatility clustering in hedge fund returns, Giamouridis & Vrontos (2007) show that the use of multivariate conditional volatility models improves portfolio performance and provides a more accurate tool for tail-risk measurement. Harris & Mazibas (2010) provide further evidence on the use of multivariate conditional volatility models in the context of dynamic hedge fund risk measurement and portfolio allocation, and show that simple volatility models, such as the

RiskMetrics EWMA model of JP Morgan, provide the biggest improvements in performance. The non-normality in hedge fund returns has prompted the use of alternative measures of risk in the optimization framework. Agarwal & Naik (2004) and Giamouridis & Vrontos (2007) compare mean-variance and mean-CVaR portfolios constructed using HFR hedge fund strategy indices. Krokmal *et al.* (2003) compare the CVaR and CDaR approaches for minimum risk portfolios of individual hedge funds, while Hentati *et al.* (2010) compare the CVaR and Omega approaches. These alternative approaches rely on non-parametric estimation, in which the moments and quantiles of the density function of portfolio returns are estimated empirically, and these are used to compute the various risk measures used in the optimization process. The non-parametric approach, while straightforward to implement, relies on a large data sample to generate sufficiently accurate estimates of the various measures. Moreover, it does not readily lend itself to incorporating the well established dynamic characteristics of hedge fund returns, such as autocorrelation and volatility clustering.

In this paper, we propose a semi-parametric approach to hedge fund portfolio optimization that addresses the shortcomings of the non-parametric approach. In the semi-parametric approach, we first standardize the returns of each portfolio constituent in order to filter out the predictable dynamics related to autocorrelation and volatility clustering. We then model the marginal density of each standardized return series using a combination of extreme value theory (for the tails of the density) and a piecewise polynomial (for the centre of the density), and construct the joint density of hedge fund index returns using a copula-based approach. We then simulate hedge fund returns from this joint density in order to compute the relevant moments and quantiles required for portfolio optimization. Using monthly index return data from the HFR database for the period 1990 to 2011, we use the semi-parametric and non-parametric approaches to obtain the optimal portfolios in the CVaR, CDaR and Omega frameworks. We compare the performance of the different estimation approaches for both conservative (i.e. minimum risk) and aggressive (i.e. maximum return) investors. For the aggressive investment strategy, we consider three different formulations of the optimization problem, each representing a different portfolio on the efficient frontier: minimization of risk subject to a target return, maximization of return subject to a risk constraint, and maximization of return per unit of risk. We also compare the semi-parametrically and non-parametrically estimated CVaR, CDaR and Omega models with a number of commonly used benchmarks, including a naïve (i.e. equally weighted) portfolio, a constant volatility mean-

variance model, a time-varying volatility mean-variance model, and a benchmark fund of hedge funds index. We report two main findings. The first is that the CVaR, CDaR and Omega optimization models offer a significant improvement in terms of risk-adjusted portfolio performance over the mean-variance optimization model and other benchmarks. The second is that semi-parametric estimation of the CVaR, CDaR and Omega models offers a very substantial improvement over non-parametric estimation. Our results are robust to the choice of target return, risk limit and estimation sample size.

The outline of the paper is as follows. Section 2 describes the optimization framework, the estimation methods and the evaluation criteria. Section 3 describes the data used in the empirical analysis. Section 4 reports the empirical results. Section 5 provides a summary and some concluding remarks.

2. Methodology

In this section, we first set out the generic optimization problem for two types of investor: conservative and aggressive, and define the different measures of risk that we use to construct the objective function in the optimization problem. We then describe the semi-parametric and non-parametric approaches to estimating the optimal portfolio in each case. Finally we define the evaluation criteria that are used to compare the different models.

2.1. Optimization Framework

Consider an investor who allocates their wealth among m individual hedge funds or investible hedge fund indices, with portfolio weight vector $\mathbf{x} = [x_1, \dots, x_m]'$. For a conservative investor, the portfolio optimization problem is given by

$$\min_{\mathbf{x}} F_p(\mathbf{x}) \tag{1}$$

$$\text{subject to } \mathbf{x} \geq 0, \mathbf{x}'\mathbf{1}_m = 1$$

where $F_p(\mathbf{x})$ is the portfolio risk function to be minimized (i.e. standard deviation, CVaR, CDaR or lower partial moment) and $\mathbf{1}_m$ is an m -vector of ones. The budget constraint and non-negativity constraints yield an unleveraged long-only portfolio. For an aggressive investor, we consider three separate formulations of the portfolio optimization problem. The first minimizes portfolio risk subject to a target expected portfolio return, r_0 :

$$\min_{\mathbf{x}} F_p(\mathbf{x}) \quad (2)$$

$$\text{subject to } \mathbf{x} \geq 0, \mathbf{x}'\mathbf{1} = 1, E(r_{p,t}) \geq r_0$$

where $\mathbf{r}_t = [r_{1,t}, \dots, r_{m,t}]'$ is the m -vector of hedge fund returns at time t and $r_{p,t} = \mathbf{x}'\mathbf{r}_t$ is the portfolio return at time t . The second formulation of the portfolio optimization problem for an aggressive investor maximizes portfolio return subject to a portfolio risk constraint:

$$\begin{aligned} \max_{\mathbf{x}} E(r_{p,t}) \\ \text{subject to } \Phi_p(\mathbf{x}) \leq \omega C, \mathbf{x} \geq 0, \mathbf{x}'\mathbf{1} = 1 \end{aligned} \quad (3)$$

where ω is the risk limit and C is the invested capital, which is set arbitrarily to 1. The third formulation of the portfolio optimization problem for an aggressive investor maximizes portfolio expected excess return per unit of portfolio risk:

$$\begin{aligned} \max_{\mathbf{x}} \frac{E(r_{p,t}) - r_f}{\Phi_p(\mathbf{x})} \\ \text{subject to } \mathbf{x} \geq 0, \mathbf{x}'\mathbf{1} = 1 \end{aligned} \quad (4)$$

where r_f is risk-free rate of return. This formulation is a generalization of the tangency portfolio in expected return-risk space for the different risk measures.

2.2. Optimization Models

We now define the different risk measures, F_p , that are used in the optimization problems described above.

2.2.1. Mean-Variance Optimization Model

As a benchmark, we use the standard mean-variance model, in which the risk measure is portfolio standard deviation, given by

$$\sigma(\mathbf{x}) = \sqrt{\mathbf{x}'\mathbf{H}\mathbf{x}} \quad (5)$$

where $\mathbf{H}$ is the $m \times m$ covariance matrix of hedge fund index returns. We consider two

versions of the mean-variance model. In the static mean-variance model, $\mathbf{H}$ is estimated using the sample covariance matrix. In the dynamic mean-variance model, $\mathbf{H}$ is estimated using the multivariate RiskMetrics EWMA model of JP Morgan (1996). For the mean-variance optimization model, we set $\Phi(\mathbf{x}) = \sigma(\mathbf{x})$.

2.2.2. Mean-CVaR Optimization Model

Conditional Value at Risk (CVaR) is a tail-risk measure derived from the Value at Risk (VaR) estimator, and is defined as the conditional expectation of losses exceeding VaR at specified confidence level and time horizon. CVaR can be defined as

$$CVaR_{\alpha}(\mathbf{x}, Z) = Z + \frac{1}{1 - \alpha} E[\xi - r_{p,t} - Z | r_{p,t} < Z] \quad (6)$$

where ζ is portfolio VaR measured at the α -confidence level, $F(\mathbf{x}, Z) = P[\xi - r_{p,t} \leq Z]$ is the cumulative distribution function, which is continuous and non-decreasing with respect to Z , and $\zeta_{\alpha}(\mathbf{x}) = \min_{\zeta} [F(\xi - r_{p,t} \leq \zeta) \geq \alpha]$. In equation (6), by minimizing CVaR, VaR is also minimized. Rockafeller & Uryasev (2002) show that CVaR satisfies all statistical axioms of a coherent measure of risk defined by Artzner *et al.* (1999) and is a convex function of portfolio positions (see also Rockafeller & Uryasev, 2000). For portfolio implementations of CVaR measure, see, for example, Krokmal *et al.* (2002a, 2002b). For the mean-CVaR optimization model, we set $\Phi(\mathbf{x}) = CVaR_{\alpha}(\mathbf{x}, \zeta)$.

2.2.3. CDaR Optimization Model

Drawdown, also known as the underwater portfolio level, is defined as the drop in portfolio value from a previous highest level. The drawdown measure helps investors to construct portfolios that may enable them not to lose more than a fixed percentage of the maximum value of their wealth achieved up to that point in time. Chekhlov *et al.* (2000) propose Conditional Drawdown at Risk (CDaR), which combines the drawdown concept with CVaR approach. Similar to CVaR, CDaR is defined as the expectation of drawdowns that exceed a certain threshold $\zeta_{\alpha}(\mathbf{x})$ defined at an α -confidence level. However, unlike CVaR, CDaR accounts not only for the amount of losses over some period, but also for the sequence of those losses. For portfolio implementation of CDaR, see, Chekhlov *et al.* (2005).

Let the uncompounded cumulative portfolio value at time t be $w(\mathbf{x}, t)$. The drawdown function at time t is given by

$$f(\mathbf{x}, t) = \max_{0 \leq j \leq t} [w(\mathbf{x}, j) - w(\mathbf{x}, t)] \quad (7)$$

We, then formulate α -CDaR as

$$CDaR_{\alpha}(\mathbf{x}, \zeta) = \zeta + \frac{1}{1-\alpha} E [f(\mathbf{x}, t) - \zeta | f(\mathbf{x}, t) > \zeta] \quad (8)$$

The drawdown function satisfies Artzner's nonnegativity, positive homogeneity, translation invariance and subadditivity axioms (see Chekhlov *et al.*, 2005). Unlike CVaR, which is estimated for a one-month time horizon, CDaR is estimated for a one-year time horizon. For the mean-CDaR optimization model, we set $\Phi(\mathbf{x}) = CDaR_{\alpha}(\mathbf{x}, \zeta)$.

2.2.4. Omega Optimization Model

Omega is a performance measure that was first introduced by Keating & Shadwick (2002a, 2002b) to overcome the shortcomings of classical measures such as the Sharpe ratio. In particular, Omega separately considers gains and losses without reference to a specific distribution for asset returns. For its implementation in portfolio construction see, for example, Avouyi-Dovi *et al.* (2004), Passow (2005), Gilli *et al.* (2006), Mausser *et al.* (2006) and Kane *et al.* (2009). Omega is defined for any portfolio return level as the probability weighted ratio of gains to losses relative to a threshold return defined by the investor, r_b .

In its simplest form, the Omega function can be expressed by the help of partitioning the portfolio return distribution into the upper partial moment of returns (gains) and the lower partial moment of returns (losses). Omega is then defined as

$$\Omega_p(\mathbf{x}, r_b) = \frac{E [r_{p,t} | r_{p,t} \geq r_b]}{r_b - E [r_{p,t} | r_{p,t} \leq r_b]} \quad (9)$$

where upper partial moments function is the conditional expectation of portfolio returns that exceed the threshold (nominator), and the lower partial moments function is the conditional expectation of returns below the threshold (denominator). For the Omega-based optimization model, we consider three specifications based on (9). For the conservative investment

strategy, (1), and the first formulation of the aggressive investment strategy, (2), we employ the lower partial moment of returns as the measure of risk and so set $F(\mathbf{x}) = r_b - E[\hat{r}_{p,t} | r_{p,t} \leq r_b]$, where the threshold return level, r_b is set to the risk-free rate of return. For the second formulation of the aggressive portfolio, (3), we maximize the upper partial moment, $g = E[\hat{r}_{p,t} | r_{p,t} \geq r_b] - r_b$, subject to a constraint on the lower partial moment, $l = r_b - E[\hat{r}_{p,t} | r_{p,t} \leq r_b]$. For the third formulation of the aggressive investment strategy, we maximize the Omega ratio as defined in (9).

2.3. Estimation Method

Hedge fund returns typically exhibit significant negative skewness and excess kurtosis, and often also display autocorrelation. Portfolio optimization in the presence of these statistical properties generally leads to very different portfolio allocations than those implied by mean-variance analysis. This has prompted the use of the alternative risk and performance measures described in the previous section, namely CVaR, CDaR and Omega. These approaches are usually implemented using a non-parametric approach in which the unobserved moments and quantiles of the distribution of portfolio returns are estimated by their empirical counterparts. The non-parametric approach, while straightforward to implement, relies on a large data sample to generate sufficiently accurate estimates of the various measures. Moreover, it does not lend itself to incorporating the well established dynamic characteristics of hedge fund returns, such as autocorrelation and volatility clustering. In this paper, we propose a semi-parametric approach to optimization in the CVaR, CDaR and Omega frameworks that addresses these shortcomings of the non-parametric approach.

First, we filter the returns for each hedge fund index using an AR(1)-EGARCH(1,1) model in order to generate an *i.i.d.* standardized return series. Second, we apply Extreme Value Theory to the standardized returns, employing the Peaks-over-Threshold approach.¹ To do so, we isolate the upper and lower tails and the centre of the standardized return distribution, and fit a Generalized Pareto Distribution to the tails of the cumulative distribution (CDF) function of

¹ In Extreme Value Theory, there are two approaches to modelling extreme values: Block-Maxima and Peaks-over-Threshold. In the Block-Maxima approach, the largest observations are collected from large samples of identically distributed observations, and the Generalized Extreme Value Distribution is fitted to the maxima of each block. In the Peaks-over-Threshold approach, a Generalized Pareto Distribution is fitted to all large observations that exceed a high threshold. Empirically, the Peaks-Over-Threshold approach appears to yield a more accurate description of the tails of the distribution for financial returns (see, for example, Embrecht *et al.*, 1997; Reiss & Thomas, 1997).

the standardized returns. To capture the centre of the return distribution, we fit a piecewise polynomial. The resulting piecewise distribution allows interpolation within the interior of the CDF and extrapolation in each tail. The ability to extrapolate the tails of the CDF allows us to estimate quantiles that lie outside the range of historical observations, and is hence particularly suited to estimating tail-related risk measures. Third, we use a copula function to model the dependency structure of individual hedge fund return indices. The most commonly used copulae are the Gaussian copula for linear correlation, the Archimedean copula and t -copula for tail-dependence, and the Gumbel copula for extreme distributions (see, for example, Bouye *et al.*, 2000). Since the AR(1)-EGARCH(1,1) filtered returns are *i.i.d.*, the t -copula is most suitable for modeling the dependency in returns. In fitting the copula function, we use both a constant correlation matrix and a dynamic correlation matrix that is obtained from the DCC-GARCH model of Engle & Sheppard (2001) and Engle (2002). Fourth, we use the resulting multivariate return distribution to simulate portfolio returns. Using the parameters of the fitted t -copula, we simulate jointly dependent hedge fund index returns by first simulating the corresponding dependent standardized returns. To do so, we first simulate dependent uniform variates, then extrapolate into the GPD tails and interpolate into the smoothed interior. We transform the uniform variates to the standardized residuals using the inversion of the semi-parametric marginal CDF of each index. Then using these simulated standardized returns as the *i.i.d.* input process, we reintroduce the autocorrelation and heteroskedasticity that is estimated using the AR(1)-EGARCH(1,1) model in the first step. Finally, we use the simulated returns of each index to estimate the optimal portfolios in the CVaR, CDaR and Omega frameworks. We also report optimization results using the non-parametric, empirically-based approach. We initially estimate each optimization model using observations $t = 1, \dots, \tau$ to generate one month-ahead out-of-sample forecast portfolio weights for month $t = \tau + 1$. The estimation sample is then rolled forward one month, and the forecast portfolio weights for month $t = \tau + 2$ are generated. The last iteration uses the sample $t = T - \tau - 1, \dots, T - 1$ to generate forecast portfolio weights for the month $t = T$. Expected returns are estimated using the corresponding sample mean over the estimation sample. The optimization models are estimated using the Matlab *fmincon* function.

2.4. Evaluation

We rebalance each portfolio at each month and calculate the realized portfolio return at the rebalancing date. We evaluate the performance of each portfolio over the out-of-sample

period $t = \tau + 1, \dots, T$ using the following metrics:

(1) *Average Realized Portfolio Return*

$$\bar{r}_P = \frac{1}{T - \tau} \sum_{t=\tau}^{T-1} \sum_{k=1}^m x_{k,t} r_{k,t+1} \quad (10)$$

(2) *Standard Deviation of Realized Portfolio Returns*

$$\sigma_P = \sqrt{\frac{1}{T - \tau} \sum_{t=\tau+1}^T \sum_{k=1}^m (x_{k,t} r_{k,t+1} - \bar{r}_P)^2} \quad (11)$$

(3) *Maximum Drawdown*

$$MDD_P = \max_{\tau \leq t \leq T} \left[\max_{\tau \leq j \leq T} w_j - w_t \right] \quad (12)$$

where w_t is the uncompounded cumulative portfolio value at date $t = \tau + 1, \dots, T$, and at $t = \tau$ initial portfolio value is set equal to 1.

(4) *CVaR*

$$CVaR_{p,t+1}^\alpha = \zeta + \frac{1}{1-\alpha} \frac{1}{T-\tau} \sum_{t=\tau}^{T-1} \max \left\{ -\sum_{k=1}^m x_{k,t} r_{k,t+1} - \zeta \right\} \quad (13)$$

where α is the CVaR confidence level and ζ is portfolio VaR.

(5) *CDaR*

$$CDaR_{p,t+1}^\alpha = \zeta + \frac{1}{1-\alpha} \sum_{j=\tau}^{T-1} \max \left\{ \max_{\tau \leq j \leq t} \left[\sum_{k=1}^m \sum_{s=\tau+1}^j x_{k,s} r_{k,s} - \sum_{k=1}^m \sum_{s=\tau+1}^t x_{k,s} r_{k,s} \right] - \zeta \right\} \quad (14)$$

where ζ is the threshold drawdown level and $CDaR_{p,t+1}^\alpha$ is portfolio CDaR defined at the α – confidence level for the out-of-sample evaluation period.

(6) *Sharpe Ratio*

$$SR_p = \frac{\bar{r}_p - r_f}{\sigma_p} \quad (15)$$

(7) *Omega Ratio*

$$Omega_p = \frac{\sum_{t=\tau}^{T-1} \max \left(\sum_{k=1}^m x_{k,t} r_{k,t+1} - r_{b,t+1}, 0 \right)}{\sum_{t=\tau}^{T-1} \max \left(r_{b,t+1} - \sum_{k=1}^m x_{k,t} r_{k,t+1}, 0 \right)} \quad (16)$$

where $r_{b,t+1}$ is the realized benchmark return at $t+1$.

(8) *Information Ratio*

$$IR_{p,t+1} = \frac{\bar{r}_p - \bar{r}_b}{\sigma_{(r_p - r_b)}} \quad (17)$$

(9) *Portfolio Turnover*

$$PT_p = \sum_{t=\tau}^{T-1} \sum_{k=1}^m |x_{k,t+1} - x_{k,t}| \quad (18)$$

3. Data

We use monthly data on ten hedge fund single strategy indices obtained from Hedge Fund Research (HFR).² These indices are convertible arbitrage (CA), distressed securities (DS), event driven (ED), equity hedge (EH), emerging markets (EM), equity market neutral (EMN), mergers arbitrage (MA), macro (MAC), relative value (RV) and short selling (SS) strategies. We use the fund of funds (FOF) strategy index as a benchmark. Our sample covers the period January 1990 to January 2011 (253 observations). This period includes a number of financial crises (e.g. Mexican crisis, Asian crisis, the Russian government default, the collapse of Long Term Capital Management, the collapse of the dotcom bubble and the most recent credit crisis). The initial estimation period is January 1990 to June 2002 (150 observations), while the out-of-sample forecast period is July 2002 to January 2011 (103 observations). Summary statistics for the hedge fund return series are reported in Table 1.

² We use the same eight HFR single strategy indices used by Giamouridis & Vrontos (2007) and Harris & Mazibas (2010) and, in addition, the short selling and emerging markets indices. HFR construct investible indices (HFRX) as counterparts to these indices (HFRI). Details can be found on the HFR website: www.hfr.com.

[Table 1]

Panel A reports various descriptive statistics for the different hedge fund strategies for the full sample of 253 observations. Unsurprisingly, hedge fund strategies exhibit quite diverse statistical characteristics. All strategies have a positive mean return. Some strategies display relatively higher volatility (such as equity hedge, emerging markets and short bias), while others have quite low volatility (such as equity market neutral, merger arbitrage, relative value). All strategies except short selling and macro exhibit negative skewness, and all are leptokurtic. The null hypothesis of normality is strongly rejected in all cases. Panel B reports the time series properties of the hedge fund return series. In particular, it reports the first five autocorrelation coefficients, the Ljung-Box portmanteau test for serial correlation up to 10 lags and the ARCH test of Engle (1982). All hedge fund strategies except short selling exhibit highly significant autocorrelations. The ARCH test suggests that there is evidence of volatility clustering in seven of the 10 strategy return series. Panel C reports pair-wise correlations between the hedge fund return series. The correlations range from -0.75 (between short bias and equity hedge) to 0.85 (between event driven and distressed securities strategy). The short selling strategy is negatively correlated with all other strategies, but otherwise the correlations are all positive. Overall, pair-wise correlations are relatively moderate, a property that is clearly desirable in the construction of funds of hedge funds.

4. Empirical Results

In this section, we report the out-of-sample performance of the different optimization approaches. In particular, we compare the non-parametric CVaR, CDaR and Omega models with the static and dynamic semi-parametric CVaR, CDaR and Omega approaches described in Section 2. We also report results for four benchmark portfolios: the static and dynamic mean-variance models, the naïve (i.e. equally weighted) portfolio, and the HFR fund of hedge funds index. We report the performance of these ten portfolios for the conservative investment strategy and the three formulations of the aggressive investment strategy. For the first formulation of the aggressive strategy (which minimizes risk subject to a target return), we set the target return equal to 14 percent. For the second formulation of the aggressive investment strategy (which maximizes return subject to a risk limit), we use risk limits of $\omega=0.02$ (monthly) for the CVaR portfolio, $\omega=0.10$ (annual) for the CDaR portfolio, and $\omega=0.0005$ for the Omega portfolio. For each portfolio, we report end of period value (assuming an initial unit investment), annualized average return, standard deviation,

maximum drawdown, CVaR, CDaR, Sharpe ratio, Omega, information ratio, and portfolio turnover. We use a risk-free rate of 2.03 percent, which is the average long term US Government bond rate for the sample period. This rate is also used as the return threshold in the calculation of Omega for optimization. In order to evaluate the sensitivity of our results to the various parameters, we consider a range of alternative return targets, risk constraints and estimation sample sizes.

Conservative Investment Strategy

Table 2 reports the results for the conservative investment strategy. Of the four benchmark portfolios, the naïve portfolio generates the highest return, but also has relatively high risk in terms of volatility, MDD, CVaR and CDaR. The dynamic MV portfolio has a slightly higher Sharpe ratio, but the Omega ratio is substantially higher for the naïve portfolio. The HFR index generates similar returns to the two MV portfolios, but it is much more volatile and, as a result, has a much lower Sharpe ratio. The non-parametric models generally underperform the four benchmarks, with none generating a return higher than that of the naïve portfolio. The Omega model, while generating the lowest returns of the three non-parametric portfolios (and lower than all four benchmark portfolios) has lower standard deviation, and very much lower MDD, CVaR and CDaR. However, it still underperforms in terms of risk-adjusted performance, with lower Sharpe and Omega ratios. A striking feature of the three non-parametric models is that they have substantially lower turnover than the static and dynamic MV benchmark portfolios, and so on balance would be preferred by investors. Overall, however, the naïve portfolio appears to offer the best investment strategy.

The static semi-parametric CVaR, CDaR and Omega models all offer returns that are higher than their non-parametric counterparts and, in all cases, higher also than the four benchmark portfolios. They are also less risky than the non-parametric models, and have higher Sharpe ratios and, for two of the three models, higher Omega ratios also. The dynamic semi-parametric CVaR, CDaR and Omega models all offer similar or better returns than the static versions with slightly higher risks and lower Sharpe ratios. However, they offer significantly higher Omega and active management ratios, and lower turnover in two out of three cases. Similar to the static models, the dynamic semi-parametric models all offer higher returns, similar or lower risks, and significantly higher Sharpe, Omega and active management ratios than both their non-parametric counterparts and the four benchmark portfolios. However, the static and dynamic semi-parametric models have substantially higher turnover than both the

non-parametric models and the two benchmark MV models, and so the superior performance of the semi-parametric models would have to be weighed against the transaction costs that they incur. Panel A of Table 6 reports the average portfolio weights of the different conservative portfolios. It is notable that the Omega-based portfolios exhibit a very different composition from the other portfolios, with a much higher allocation to risk-reducing strategies (i.e. equity market neutral, short bias, arbitrage strategies including merger, convertible and relative value arbitrage) and much lower allocation to return-enhancing strategies (i.e. event driven, distressed securities, equity hedge, emerging markets and macro).

[Table 2]

Aggressive Investment Strategy

Table 3 reports the results for the first formulation of the aggressive investment strategy, which minimizes portfolio risk subject to a target return. As expected, imposing a relatively high target return leads to portfolios that generate higher returns relative to the conservative investment strategy but which are also significantly more risky in terms of standard deviation, MDD, CVaR and CDaR. For the benchmark MV portfolios and the non-parametric portfolios, the increase in risk outweighs the increase in return, and consequently imposing a target return reduces the Sharpe ratio. However, the effect on the Omega ratio is less clear, with a decrease for the two MV portfolios and the CVaR portfolio, but an increase for the CDaR and Omega portfolios. The semi-parametric CDaR portfolio performs worse in the presence of a target return, with lower return, higher risk and a much lower Sharpe ratio, but the semi-parametric CVaR and Omega portfolios generate similar performance relative to the conservative investment strategy, in both their static and dynamic variants. The semi-parametric models clearly dominate the non-parametric models for this formulation of the aggressive investment strategy. In particular, no non-parametric model is able to match the performance of any of the semi-parametric models, static or dynamic. The imposition of a target return substantially increases portfolio turnover, especially for the semi-parametric models.

[Table 3]

Table 4 reports the results for the second formulation of the aggressive investment strategy, which maximizes portfolio return subject to a maximum risk constraint. For all portfolios, the

returns are higher than in the minimum risk formulation of the aggressive investment strategy, although the resulting portfolio risk is similar in the two cases. In terms of risk-adjusted performance, there is some improvement in both the Sharpe ratio and the Omega ratio. Transaction costs are generally lower than in the previous case. Of the four benchmark portfolios, the static and dynamic MV portfolios offer the highest return, although also higher risk. Of the non-parametric models, only the Omega model is able to match the performance of the MV benchmark portfolios. As in the previous case, both the static and dynamic semi-parametric models offer a substantial improvement in performance over their non-parametric counterparts, both in absolute return terms and in terms of risk-adjusted performance.

[Table 4]

Table 5 reports the results for the third formulation of the aggressive investment strategy, which maximizes the return per unit of risk. For the benchmark MV portfolios and the non-parametric models, the results are very similar to the conservative investment strategy, with slightly lower returns and slightly lower risk, and very similar Sharpe and Omega ratios. However, for the semi-parametric models, while returns are significantly higher, risk is lower, and so there is a significant improvement in risk-adjusted performance relative to the conservative case. Consequently, both the static and dynamic semi-parametric models comfortably outperform their non-parametric counterparts.

[Table 5]

Panels B to D of Table 6 report the average portfolio weights of the different aggressive portfolios. The semi-parametric models exhibit very different portfolio compositions compared to both the non-parametric models and the benchmark models, for all three formulations of the aggressive investment strategy. In particular, the CVaR- and Omega-based portfolios make significantly higher allocations to risk-reducing strategies and much lower allocations to return-enhancing strategies. Although, the semi-parametric CDaR-based portfolios have compositions that are similar to their non-parametric counterparts for the first formulation of the aggressive investment strategy, they make similar allocations to the CVaR- and Omega-based portfolios in the second and third formulation. The portfolio compositions suggest that the superior performance of the semi-parametric models is due to their well-balanced portfolio allocations between risk-reducing and return-enhancing strategies.

[Table 6]

Sensitivity Analysis

In order to test robustness of the findings reported above, we examine the sensitivity of the portfolio performance to the risk limits, return targets and estimation sample sizes. For the sake of brevity, we do not report the numerical results of this sensitivity analysis, but instead describe the main findings. The full results of the analysis are available from the authors on request.

First, we examine the sensitivity of the CVaR, CDaR, Omega and MV portfolios to the risk limits in the second formulation of the aggressive investment strategy. For the CVaR and CDaR models, increasing the risk limit enhances portfolio returns as well as risk. In terms of risk-adjusted returns, increasing the risk limit increases the Sharpe ratio, information ratio and Omega ratio up to a certain level. Beyond this, portfolio risk increases faster than return, and so the risk-adjusted performance declines. The semi-parametric CVaR and CDaR models significantly outperform the non-parametric CVaR and CDaR models at every level of the risk limit. At lower risk limits, the non-parametric models yield portfolios with lower risk and return. Increasing the risk limit significantly enhances the return performance of the semi-parametric models while portfolio risk remains lower than for the non-parametric models, resulting in a significant improvement in risk-adjusted return performance. In contrast with the CVaR and CDaR models, for the Omega model, increasing the risk limit results in an allocation with more portfolio weights given to instruments with higher downside potential. For the non-parametric Omega portfolios, the allocation is insensitive to the risk limit up to a very high level. However, the semi-parametric Omega models are quite sensitive to the risk limit, and yield higher return, lower risk, and hence better risk-adjusted return performance at low levels of the risk limit. Increasing the risk limit worsens the return performance while portfolio risk is consistently lower than for the non-parametric model. Moreover, the dynamic model is more sensitive than the static model to the risk limit. Both models significantly outperform the non-parametric model, especially at low levels of the risk limit. For the MV models, the performance of the static model is insensitive to changes in the risk limit, but for the dynamic model, returns increase faster than risk as the risk limit rises, and so has better risk-adjusted return performance than static model at every level of the risk limit.

Second, we employed alternative return targets between 13 and 18 percent. The use of either

lower or higher return targets in the first formulation of the aggressive investment strategy does not significantly alter our main findings. At lower return targets, MV-based models and non-parametric CDaR, CVaR and Omega models exhibit poorer return, risk and risk-adjusted return performance than the naïve portfolio. Among the semi-parametric models, the CDaR model exhibits slightly better performance than its non-parametric counterpart. However, the semi-parametric CVaR and Omega models significantly outperform the naïve portfolio in terms of return, risk and risk-adjusted returns for all the target returns considered. These models also make quite balanced allocations between risk-reducing and return-enhancing strategies, allocating half of the portfolio to risk-reducing strategies, while the other models allocate only a small fraction of the portfolio to risk-reducing strategies. At higher return targets, while other models lead to portfolios comprising solely return-enhancing strategies, the semi-parametric CVaR and Omega models still allocate a significant portion of the portfolio to risk-reducing strategies. This difference in portfolio allocation is reflected in portfolio performance. The semi-parametric CVaR and Omega models exhibit significantly higher portfolio return, lower portfolio risk and notably higher risk-adjusted return statistics. In contrast with the other models, increasing the return target further improves their portfolio performance as they generate higher portfolio returns with similar or slightly higher portfolio risk, hence higher risk-adjusted returns.

Third, we employ two shorter estimation periods (100 and 125 months) and one longer estimation period (175 months). The use of a shorter or longer estimation sample periods does not change our main findings. In particular, for all portfolios with the exception of the second formulation of the aggressive portfolio, changing the sample size simultaneously increases both the portfolio return and risk, and marginally increases the risk-adjusted return performance. For the maximum return formulation of the aggressive portfolio, reducing the sample size leads to a reduction in portfolio return performance and risk, and a marginal reduction in risk-adjusted performance.

4. Conclusion

Portfolio optimization in a hedge fund context is becoming increasingly relevant as a result of the growth in the market for investible hedge fund indices and funds of hedge funds. The non-normality in hedge fund returns has prompted the use of alternative measures of risk in the optimization framework, such as CVaR, CDaR and Omega, which capture the tail risks that are inherent in hedge fund investment. While a number of studies have examined

portfolio optimization in a hedge fund context, these studies generally rely on non-parametric estimation, in which the moments and quantiles of the density function of portfolio returns are estimated empirically, and these are used to compute the various risk measures used in the optimization process. The non-parametric approach, while straightforward to implement, relies on a large data sample to generate sufficiently accurate estimates of the various measures. Moreover, it does not readily lend itself to incorporating the well established dynamic characteristics of hedge fund returns, such as autocorrelation and volatility clustering.

In this paper, we propose a semi-parametric approach to hedge fund portfolio optimization that addresses the shortcomings of the non-parametric approach. Using monthly index return data from the HFR database for the period 1990 to 2011, we use the semi-parametric and non-parametric approaches to obtain the optimal portfolios in the CVaR, CDaR and Omega frameworks. We compare the performance of the different estimation approaches for conservative (minimum risk) as well as aggressive (maximum return) investors. We also compare the semi-parametrically and non-parametrically estimated CVaR, CDaR and Omega models with a number of commonly used benchmarks, including a naïve (i.e. equally weighted) portfolio, a constant volatility mean-variance model, a time-varying volatility mean-variance model, and a benchmark fund of hedge funds index. We report two main findings. The first is that the CVaR, CDaR and Omega optimization models offer a significant improvement in terms of risk-adjusted portfolio performance over the mean-variance optimization model. The second is that semi-parametric estimation of the CVaR, CDaR and Omega models offers a very significant improvement over non-parametric estimation. Our results are robust to as the choice of target return, risk limit and estimation period.

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Table 1:**Summary Statistics and Time Series Properties of Hedge Fund Indices**

The table (Panel A) reports summary statistics in percentages for the hedge fund strategy index series over the period of May 1998 to January 2011 (153 months). Panel B reports the autoregressive conditional heteroskedasticity (ARCH) and autocorrelation test results for the full period. The Ljung-Box-Q test for autocorrelation of order upto 10 asymptotically distributed as a central Chi-square with 10 d.o.f. under the null hypothesis, with 5% critical value 18.307. ARCH(4) is Engle's LM test for autoregressive conditional heteroskedasticity, which is asymptotically distributed as a central Chi-square with four d.o.f. under the null hypothesis with 5% critical value 9.488. *p*-values are also reported in the adjacent columns. Panel C reports HFR single strategy hedge fund index correlations.

Panel A: Summary Statistics									
Index	Mean	Median	Std. Dev.	Min.	Max.	Skew	Kurtosis	JB Stats	pval
Convertible Arbitrage	0.77	1.01	1.94	-16.01	9.74	-3.21	29.58	9657.61	0.00
Distressed Securities	1.02	1.13	1.90	-8.50	7.06	-1.03	5.00	308.06	0.00
Event Driven	1.00	1.29	1.99	-8.90	5.13	-1.36	4.25	268.98	0.00
Equity Hedge	1.13	1.30	2.65	-9.46	10.88	-0.24	1.87	39.36	0.00
Emerging Markets	1.19	1.59	4.17	-21.02	14.80	-0.89	3.73	180.22	0.00
Equity Market Neutral	0.60	0.58	0.92	-2.87	3.59	-0.13	1.17	15.09	0.01
Mergers Arbitrage	0.74	0.92	1.20	-6.46	3.12	-2.20	8.87	1032.68	0.00
Macro	1.09	0.81	2.22	-6.40	7.88	0.46	0.84	16.24	0.00
Relative Value	0.86	0.93	1.28	-8.03	5.72	-2.24	14.12	2311.63	0.00
Short Bias	0.17	-0.20	5.58	-21.21	22.84	0.18	1.93	40.78	0.00
Panel B: Basic Time Series Properties									
Index	ARCH	pval	LB-Q	pval	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)
Convertible Arbitrage	50.96	0.00	132.11	0.00	0.59	0.29	0.17	0.12	-0.02
Distressed Securities	36.49	0.00	110.14	0.00	0.55	0.27	0.16	0.14	0.02
Event Driven	6.75	0.15	47.40	0.00	0.38	0.15	0.11	0.06	0.02
Equity Hedge	23.33	0.00	27.72	0.00	0.25	0.15	0.09	0.04	-0.05
Emerging Markets	4.08	0.40	42.63	0.00	0.34	0.15	0.08	0.05	0.01
Equity Market Neutral	19.54	0.00	83.31	0.00	0.16	0.19	0.21	0.19	0.09
Mergers Arbitrage	3.29	0.51	28.33	0.00	0.22	0.08	0.11	0.00	0.07
Macro	13.90	0.01	23.07	0.01	0.16	0.00	0.01	0.11	0.17
Relative Value	31.52	0.00	74.87	0.00	0.44	0.25	0.13	0.11	0.02
Short Bias	54.35	0.00	9.63	0.47	0.10	-0.06	0.01	-0.09	-0.09
Panel C: Correlations between single hedge fund strategy indices									
Index	CA	DS	ED	EH	EM	EMN	MA	MAC	RV
Convertible Arbitrage	1.00								
Distressed Securities	0.64	1.00							
Event Driven	0.64	0.85	1.00						
Equity Hedge	0.57	0.68	0.83	1.00					
Emerging Markets	0.51	0.68	0.74	0.72	1.00				
Equity Market Neutral	0.26	0.33	0.35	0.45	0.21	1.00			
Mergers Arbitrage	0.46	0.56	0.75	0.58	0.51	0.31	1.00		
Macro	0.23	0.42	0.51	0.57	0.57	0.32	0.33	1.00	
Relative Value	0.79	0.77	0.74	0.66	0.59	0.36	0.54	0.33	1.00
Short Bias	-0.31	-0.48	-0.62	-0.75	-0.58	-0.11	-0.41	-0.36	-0.39

Table 2:
Risk, Return and Turnover Performance Measures of Conservative Portfolio

The table reports evaluation criteria for the out-of-sample monthly rebalancing conservative portfolio in the period July 2002 to January 2011 (103 months). Benchmark models are HFR Fund of Fund Index, naive (equally-weighted) portfolio, static and dynamic mean-variance (MV) models. Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99% confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Benchmark models										
HFR index	1.444	4.45	5.62	25.11	22.63	1.62	0.124	1.473	-	-
Naive	1.736	6.54	4.52	17.29	19.31	4.49	0.289	2.397	0.338	0.00
MV (static)	1.468	4.54	3.31	12.91	14.74	3.42	0.218	1.024	0.009	28.20
MV (dynamic)	1.515	4.88	2.69	5.43	11.85	1.49	0.306	1.083	0.028	24.03
Non-parametric models										
CVaR	1.698	6.27	4.22	15.88	17.65	4.15	0.290	1.994	0.266	1.04
CDaR	1.430	4.25	3.93	15.76	17.77	4.12	0.163	0.938	- 0.024	1.78
Omega	1.424	4.16	2.58	7.82	12.07	2.13	0.239	0.941	- 0.023	7.01
Semi-parametric models (static)										
CVaR	1.826	7.08	2.99	5.00	8.09	1.37	0.488	1.716	0.173	40.93
CDaR	2.019	8.30	4.08	8.31	15.15	2.25	0.444	4.521	0.382	19.18
Omega	1.926	7.72	3.37	5.10	7.57	1.40	0.487	1.898	0.202	58.87
Semi-parametric models (dynamic)										
CVaR	2.023	8.32	4.13	8.92	15.02	2.41	0.440	5.118	0.409	17.90
CDaR	1.842	7.22	3.98	9.72	10.60	2.62	0.376	2.031	0.249	63.99
Omega	1.941	7.83	4.03	7.34	14.76	2.00	0.416	2.146	0.255	57.89

Table 3:
Risk, Return and Turnover Performance Measures of Minimum Risk Aggressive Portfolio (Formulation 1)

The table reports evaluation criteria for the out-of-sample monthly rebalancing aggressive portfolio with minimum risk formulation with an annual nominal target return 14% in the period July 2002 to January 2011 (103 months). Benchmark models are HFR Fund of Fund Index, naive (equally-weighted) portfolio, static and dynamic mean-variance (MV) models. Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99% confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Benchmark models										
HFR index	1.444	4.45	5.62	25.11	22.63	1.62	0.124	1.473	0.000	0.00
Naive	1.736	6.54	4.52	17.29	19.31	4.49	0.289	2.397	0.338	0.00
MV (static)	1.688	6.51	8.79	41.68	41.26	9.64	0.147	1.630	0.149	15.97
MV (dynamic)	1.685	6.48	8.79	41.68	41.26	9.64	0.146	1.601	0.146	16.33
Non-parametric models										
CVaR	1.796	7.25	9.00	41.48	41.26	9.60	0.168	1.851	0.200	12.93
CDaR	1.715	6.70	8.91	41.54	41.26	9.61	0.151	1.679	0.162	13.27
Omega	1.643	6.18	8.63	41.50	41.26	9.60	0.139	1.520	0.130	15.25
Semi-parametric models (static)										
CVaR	2.475	10.76	5.54	8.82	12.45	2.37	0.455	3.790	0.331	112.10
CDaR	1.811	7.35	8.97	41.48	41.26	9.60	0.171	1.901	0.207	18.53
Omega	2.550	11.10	5.41	8.93	12.44	2.40	0.484	4.309	0.362	112.65
Semi-parametric models (dynamic)										
CVaR	2.489	10.83	5.64	8.81	12.45	2.37	0.450	3.936	0.335	114.55
CDaR	1.809	7.34	8.98	41.49	41.26	9.60	0.171	1.887	0.206	16.93
Omega	2.529	11.02	5.63	9.18	12.45	2.47	0.461	4.052	0.344	112.32

Table 4:
Risk, Return and Turnover Performance Measures of Maximum Return Aggressive Portfolio (Formulation 2)

The table reports evaluation criteria for the out-of-sample monthly rebalancing aggressive portfolio with maximum return formulation with a risk constraint in the period July 2002 to January 2011 (103 months). Benchmark models are HFR Fund of Fund Index, naive (equally-weighted) portfolio, static and dynamic mean-variance (MV) models. Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99% confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Benchmark models										
HFR index	1.444	4.45	5.62	25.11	22.63	1.62	0.124	1.473	0.000	0.00
Naive	1.736	6.54	4.52	17.29	19.31	4.49	0.289	2.397	0.338	0.00
MV (static)	1.819	7.43	9.30	41.57	41.18	9.62	0.168	1.768	0.197	13.63
MV (dynamic)	2.184	9.61	9.67	40.54	41.17	9.42	0.226	2.352	0.323	9.98
Non-parametric models										
CVaR	1.675	6.13	4.54	14.05	16.44	3.71	0.261	1.888	0.224	18.88
CDaR	1.818	7.42	9.33	41.63	41.26	9.63	0.167	1.757	0.195	14.00
Omega	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.222	0.302	0.00
Semi-parametric models (static)										
CVaR	3.363	14.40	6.28	6.14	8.33	1.68	0.569	9.055	0.563	99.95
CDaR	3.375	14.53	7.57	15.89	17.28	4.15	0.477	6.836	0.608	97.35
Omega	2.903	12.91	9.32	32.48	30.48	7.84	0.337	3.925	0.499	57.00
Semi-parametric models (dynamic)										
CVaR	3.187	13.74	5.83	5.60	8.37	1.54	0.580	13.850	0.698	98.42
CDaR	3.505	14.98	7.53	10.18	12.46	2.74	0.496	6.467	0.609	98.90
Omega	2.990	13.14	8.00	12.41	13.98	3.30	0.401	3.320	0.420	80.00

Table 5:
Risk, Return and Turnover Performance Measures of Maximum Excess Return over Minimum Risk Aggressive Portfolio (Formulation 3)

The table reports evaluation criteria for the out-of-sample monthly rebalancing aggressive portfolio with maximum excess return over minimum risk formulation in the period July 2002 to January 2011 (103 months). Benchmark models are HFR Fund of Fund Index, naive (equally-weighted) portfolio, static and dynamic mean-variance (MV) models. Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99% confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Benchmark models										
HFR index	1.444	4.45	5.62	25.11	22.63	1.62	0.124	1.473	0.000	0.00
Naive	1.736	6.54	4.52	17.29	19.31	4.49	0.289	2.397	0.338	0.00
MV (static)	1.452	4.40	2.96	9.43	14.02	2.54	0.231	0.988	- 0.005	10.28
MV (dynamic)	1.490	4.68	2.17	2.72	5.98	0.76	0.352	1.041	0.014	21.59
Non-parametric models										
CVaR	1.426	4.17	2.69	4.87	7.92	1.34	0.230	0.946	- 0.019	15.01
CDaR	1.462	4.47	2.78	6.22	9.13	1.70	0.254	1.006	0.002	11.26
Omega	1.464	4.49	3.06	9.95	14.16	2.68	0.232	1.012	0.005	10.21
Semi-parametric models (static)										
CVaR	2.027	8.32	3.40	4.26	5.45	1.17	0.534	2.135	0.230	70.00
CDaR	2.060	8.52	3.93	5.91	10.78	1.61	0.477	2.641	0.276	62.60
Omega	2.115	8.82	3.56	4.52	5.70	1.24	0.551	2.462	0.264	74.42
Semi-parametric models (dynamic)										
CVaR	2.332	9.98	4.01	5.16	6.93	1.41	0.573	3.440	0.352	66.43
CDaR	2.067	8.59	4.55	12.50	12.59	3.32	0.417	3.197	0.408	70.84
Omega	2.461	10.63	4.45	5.57	7.15	1.52	0.559	4.063	0.368	80.92

Table 6a:
Portfolio Composition with 1-Month Rebalancing Portfolios

Panel A and Panel B report average portfolio weights of the ten single strategy hedge fund indices for conservative portfolio and aggressive portfolio with minimum risk formulation with an annual nominal target return 14% in the out-of-sample period July 2002 to January 2011. Maximum position for each asset is limited to 50% of the portfolio to force diversification. CVaR and CDaR portfolios are calculated at 95% confidence level.

	CA	DS	ED	EH	EM	EMN	MA	MAC	RV	SB
Panel A: Conservative Portfolio										
Benchmark models										
MV (static)	12.00	6.82	5.82	7.57	3.20	20.11	13.57	7.23	12.14	11.54
MV (dynamic)	8.19	7.01	6.27	6.27	3.60	16.57	13.43	7.36	12.22	19.08
Non-parametric models										
CVaR	9.89	9.73	9.71	9.80	9.03	10.31	10.12	10.02	10.03	11.37
CDaR	11.87	5.51	6.81	8.65	2.03	17.50	15.92	15.49	9.65	6.57
Omega	8.05	2.93	0.00	8.66	0.01	26.62	21.95	3.11	18.45	10.23
Semi-parametric models (static)										
CVaR	9.00	10.11	4.50	8.28	2.80	23.81	8.85	7.50	10.82	14.33
CDaR	9.97	10.70	8.03	10.30	5.89	14.36	11.50	12.28	11.76	5.21
Omega	8.34	10.48	1.47	7.08	1.45	31.80	6.83	6.25	15.64	10.67
Semi-parametric models (dynamic)										
CVaR	9.94	9.28	2.25	2.68	0.79	38.76	5.22	11.44	18.70	0.95
CDaR	9.94	10.70	8.97	10.03	5.89	14.77	11.01	12.81	11.86	4.03
Omega	9.36	10.25	1.26	0.64	0.00	38.81	5.80	7.68	26.20	0.00
Panel B: Minimum Risk Aggressive Portfolio (Formulation 1)										
Benchmark models										
MV (static)	0.58	5.02	18.66	46.46	17.37	0.15	0.37	5.92	1.67	3.81
MV (dynamic)	0.96	5.54	12.66	46.14	17.86	0.27	0.30	8.78	4.80	2.70
Non-parametric models										
CVaR	1.86	3.43	14.78	41.48	24.36	1.41	1.77	7.89	2.08	0.92
CDaR	2.34	3.07	15.81	42.63	19.50	1.76	2.14	9.79	2.40	0.57
Omega	0.09	5.67	16.73	48.23	15.78	0.05	0.00	6.66	2.61	4.18
Semi-parametric models (static)										
CVaR	11.97	17.30	6.85	15.38	7.89	5.50	6.23	12.81	6.17	9.89
CDaR	1.95	4.17	13.56	42.54	23.05	1.79	1.78	7.53	3.07	0.56
Omega	12.14	17.34	6.57	14.26	8.11	6.04	6.49	12.41	6.83	9.82
Semi-parametric models (dynamic)										
CVaR	11.30	19.46	6.73	10.71	6.04	9.34	5.56	12.73	13.17	4.97
CDaR	2.06	4.18	14.37	42.29	22.61	1.76	1.81	7.71	2.69	0.52
Omega	11.12	19.43	6.61	9.26	6.30	8.72	6.67	12.12	15.43	4.34

Table 6b:
Portfolio Composition with 1-Month Rebalancing Portfolios (cont'd)

Panel C and Panel D report average portfolio weights of the ten single strategy hedge fund indices for maximum return aggressive portfolio with a risk limit portfolio (Formulation 2) and maximum excess return over minimum risk aggressive portfolio (Formulation 3) in the out-of-sample period July 2002 to January 2011. Maximum position for each asset is limited to 50% of the portfolio to force diversification. CVaR and CDaR portfolios are calculated at 95% confidence level.

	CA	DS	ED	EH	EM	EMN	MA	MAC	RV	SB
Panel C: Maximum Return Aggressive Portfolio (Formulation 2)										
Benchmark models										
MV (static)	0.03	0.13	12.19	49.29	25.71	0.02	0.03	12.55	0.04	0.01
MV (dynamic)	0.02	0.09	1.74	48.83	42.42	0.01	0.02	6.83	0.02	0.01
Non-parametric models										
CVaR	4.75	10.75	10.22	24.72	4.39	4.99	6.39	18.44	5.99	9.37
CDaR	0.00	0.01	12.15	49.50	25.72	0.00	0.00	12.61	0.00	0.00
Omega	0.00	0.00	0.00	50.00	50.00	0.00	0.00	0.00	0.00	0.00
Semi-parametric models (static)										
CVaR	12.53	21.49	4.04	15.43	17.98	2.70	4.15	14.06	3.65	3.98
CDaR	7.35	14.16	18.02	23.86	23.87	0.99	0.09	7.91	2.99	0.76
Omega	4.37	6.31	19.42	21.36	42.72	0.00	0.00	5.34	0.49	0.00
Semi-parametric models (dynamic)										
CVaR	12.23	23.34	4.74	13.40	13.32	7.10	4.09	15.25	5.85	0.67
CDaR	7.38	15.15	16.62	16.12	28.26	1.26	0.26	11.10	1.56	2.30
Omega	4.85	3.40	16.50	12.62	41.75	0.00	0.00	8.25	0.00	12.62
Panel D: Maximum Excess Return over Minimum Risk Aggressive Portfolio (Formulation 3)										
Benchmark models										
MV (static)	11.71	6.56	0.43	12.23	0.08	12.82	18.42	7.67	19.68	10.39
MV (dynamic)	3.30	8.39	2.21	5.98	0.06	16.69	20.51	5.72	23.02	14.13
Non-parametric models										
CVaR	14.07	0.00	0.32	23.96	0.00	16.74	17.65	10.64	1.39	15.23
CDaR	9.71	0.90	1.93	1.33	0.47	39.17	25.17	15.65	4.29	1.39
Omega	6.45	4.04	0.00	17.51	0.00	11.50	17.22	6.91	23.93	12.44
Semi-parametric models (static)										
CVaR	9.22	11.64	1.42	8.37	1.57	28.40	5.93	9.60	12.85	11.00
CDaR	12.17	9.96	1.88	8.97	0.25	34.42	7.25	9.62	13.96	1.52
Omega	9.61	11.62	2.47	10.45	1.56	24.18	6.73	8.59	13.26	11.53
Semi-parametric models (dynamic)										
CVaR	11.14	15.86	1.73	2.73	0.02	33.21	3.52	14.17	17.31	0.32
CDaR	13.64	15.97	2.49	8.97	0.30	29.88	5.66	9.74	13.05	0.33
Omega	12.10	14.89	2.54	3.23	0.89	24.25	6.04	10.10	25.44	0.52