Liability law under scientific uncertainty

Luigi Alberto Franzoni∗†

March 2017

Abstract

This paper investigates the implications of uncertainty aversion on optimal liability law. Of special interest is the case in which the causal link between conduct and harm is not known with certainty, as is frequently the case with toxic torts. Under negligence, uncertainty aversion calls for a higher standard of care if, and only if, the safest prevention measures are also the most reliable ones (i.e., they reduce the uncertainty perceived by the victim). Strict liability dominates negligence when the injurer has lower degrees of uncertainty aversion than the victim and can formulate more precise estimates of the probability of harm. When harm is dispersed on a very large number of victims, however, negligence dominates independently of their degree of uncertainty aversion.

Keywords: negligence vs. strict liability, scientific uncertainty, ambiguity, toxic torts, Hand’s rule.

JEL Code: K13

∗Correspondence to: Luigi A. Franzoni, Department of Economics, Piazza Scaravilli 2, 40126 Bologna, Italy.

†This paper stems from a larger project originally titled “Liability law and uncertainty spreading.” I am grateful to Fabio Maccheroni and Massimo Marinacci for valuable help. Thanks also to two anonymous referees, S. Bose, A. Daughety, C. Engel, M. Faure, G. Dari Mattiacci, D. Heine, E. Langlais, J. Reinganum, F. Parisi, S. Romagnoli, S. Shavell, O. Somech, L. Visscher, A. Tabbach, U. Schweizer, and seminar participants in Bologna, Bonn, Ghent, Istanbul, Santo Domingo (ALACDE), Stockholm (EALE), Standford (ALEA), and Tel Aviv for constructive comments.
“[A]rguably, there are no certainties in science.”

The Supreme Court in Daubert v. Merrel Dow Pharmaceuticals, Inc. (1993).

1 Introduction

Liability rules allocate risks across individuals. As such, they serve both an incentive function (encouraging parties to take care) and an insurance function (mitigating the loss for the victims). Optimal liability law design accordingly involves a difficult trade-off, in which the ability of the parties to take measures to avoid harm is balanced against their ability to bear uncertainty.

In the ’60s, product liability was introduced on the assumption that manufacturers were in the best position to shoulder the risks associated with mass production.¹ The risks of ’60 were to a large extent predictable ones, as they were associated with simple products (like cars and home appliances) causing, with some calculable probability, immediate harm. In contrast, many hazards of the current time tend to have an “invisible” nature: the harmful effects of new substances and agents tend to materialize after long latency periods. Furthermore, the probability by which these harmful effect are likely to occur is itself hard to establish, as the causal links between exposure and disease development are complex and not well understood. Very often, different causal models are advanced in the scientific community, and all models come with some degree of plausibility. In these cases, the uncertainty allocated by the liability system differs from standard calculable risk, as it extends into the realm of scientific uncertainty.²

¹As famously argued by Judge Trayor in Escola v. Coca-Cola Bottling Co., 24 Cal.2d 453, 150 P.2d 436 (1944). This view, rooted in the “enterprise liability” theory, is still shared by many courts and commentators (see Owen (2014), §5.4). Whether manufacturers are indeed in the best position to provide insurance to consumers against risk, is at the center of the current debate on liability reform. See, for instance, Polinsky and Shavell (2010), and Hersch and Viscusi (2013).

²Junk science is obviously excluded, comporting with Daubert v. Merrel Dow Pharmaceuticals, Inc. 509, U.S. 579 (1993). Scientific uncertainty has been a distinguishing trait of leading toxic torts, including asbestos, Agent Orange, Dalkon Shield, DES, and Bendectin (see, for instance, Geistfeld (2001)). Sometimes uncertainty concerns the relationships between the extent of exposure and occurrence of harm, as in the case of the controversial “single fibre” theory for the insurgence of asbestos diseases (see Moeller v. Garlock Sealing Technologies, LLC, 660 F.3d 950, (6th Cir. 2011)).
The massive dose of uncertainty brought about by the lack of knowledge poses a new challenge to liability law and safety policy in general (see Institute of Medicine (2014)).

In this article, I investigate the implication of both standard risk and scientific uncertainty on optimal liability law. I addresses two main issues. The first concerns the impact of uncertainty on the optimal structure of negligence law and strict liability. Of special interest in the question whether uncertainty about the actual degree of hazardousness of an activity calls for a cautionary approach in the setting of the standard of care. The second issue concerns the choice between liability rules: is negligence (where the standard is set by the lawmaker) preferable to strict liability (where the level of care is decided by the injurer)? Should manufacturers still be regarded as the best uncertainty bearers, when uncertainty stems from the lack of scientific knowledge rather than the known risks of mass production?

In the model, the probability that an activity causes harm may not be known with certainty. For example, this probability might belong to an interval (say, the probability of harm lies between 2% and 10%) or it might take two values (expert A says the probability is 2%, expert B says it 10%). Parties formulate beliefs about the probability of harm and attach a credibility weight to these beliefs.

I assume that the beliefs of the parties are correct on average, so as to reflect the foreseeability of harm. If parties know with certainty the probability of harm (beliefs are concentrated at one number), the model comports with the standard expected utility approach. If beliefs are dispersed, the decision environment includes “ambiguity.” In line with Ellsberg (1961) and an impressive body of empirical evidence, I assume that

3With uncertainty are fraught also a very large number of chemical compounds - with about 6,000 chemicals of concern for exposure (Cranor (2011)) - and GMOs (Strauss (2012)). While the applications of nanotechnology grow at an exponential rate (more than 1,800 nano-influenced consumer products are present in the marketplace), so do the highly unknown risks from exposure to nano fibers (see David (2011)). Uncertainty surrounds, however, also older technologies, like nuclear energy production.

4The cautionary approach permeates European safety policy (under the so called Precautionary Principle). US law makers and regulators prefer to avoid the (vague) language of the Precautionary Principle and stick to standard cost-benefit analysis. In practice, however, they are believed to pursue a cautionary approach, especially when evidence is scarce (see, for instance, Sunstein (2005), and Viscusi and Zeckhauser (2015)). In risk assessment, a cautionary approach underpins sensitivity analysis.
people are averse both to risk and ambiguity.\(^5\) Ambiguity aversion is modelled according to the smooth model of Klibanoff et al. (2005), which posits that parties are averse to mean preserving spreads of the beliefs (so, to know that the probability of harm is 6\% for sure is better than knowing that it might be either 2\% or 10\%).

To obtain simple results, I rely on a local approximation of the parties’ willingness to bear uncertainty. In Section 3, I show that, for small losses, the decision problem takes a simple mean-variance shape, in which the premium that an individual is willing to pay to avoid uncertainty is the sum of a risk premium and an ambiguity premium. The risk premium depends on the variance of the mean probability of harm, while the ambiguity premium depends on the variance of the beliefs, as in Maccheroni et al. (2013). This formulation is particularly suitable for policy analysis, since it yields prescriptions that are independent of income levels. Specifically, this convention comports both with current liability law and standard cost-benefit analysis.

Uncertainty aversion introduces a new dimension in the evaluation of precautionary measures. These should be assessed against two factors: i) their ability to reduce expected harm (“safety”), and ii) the accuracy by which they can do so (“reliability”). A marginally “safer” precautionary measure might be discarded, if its effects are highly uncertain. For instance, a new drug might have better expected therapeutic effects than an older one, but these effects might be highly debated. If parties are highly averse to uncertainty, they will opt for the old drug. The trade-off between safety and reliability faced by liability law is at the center of this contribution.

Section 4 deals with the optimal design of liability law.\(^6\) In an ideal world, in which the law-maker can control both the level of care taken by the injurer and the allocation

\(^5\) Risk and ambiguity aversion are the cornerstones of modern decision theory (see, for instance, Wakker (2010)). Risk and ambiguity aversion are particularly pronounced for low probability losses, the typical case in torts. Interestingly, also chimpanzees and bonobos appear to dislike risk and, to a greater extent, ambiguity (see Rosati and Hare (2011)).

\(^6\) This paper studies the impact of uncertainty on the allocation of the loss in the traditional strict liability vs. negligence set-up (see Shavell (2007)). An alternative interpretation is to assume strict liability, and to consider the impact of uncertainty on the causation standard. If the causation standard is not met, the loss falls on the victims.
of the loss between the parties (full efficiency), a dollar invested in prevention should reduce expected harm and the uncertainty burden for the parties by one dollar. Each party should bear a share of the loss proportional to his/her tolerance to uncertainty (see Appendix A3). Neither strict liability nor negligence produce the fully efficient outcome.⁷

Under strict liability, the injurer decides the level of care to take, whereas the courts set the damage awards. Damages can fall short of full compensation for victims because of statutory caps or explicit exclusion of certain harms; however, victims are actually overcompensated in cases in which the courts award punitive damages. I show that optimal uncertainty sharing requires damages to be less than fully compensatory: some uncertainty should be borne by the victims too. Optimal damages increase with the degree of risk aversion of the victims and, if safety and reliability go hand in hand (i.e., if care reduces ambiguity), with their degree of ambiguity aversion.

Under the negligence rule, the injurer bears the loss only if she does not meet the standard of care. If she does, no damages are awarded (so, there is no uncertainty sharing). The standard of care is set by the courts, which balance the costs and benefits of prevention.⁸ The optimal standard of care increases with the degree of risk aversion of the victims, while it increases with their degree of ambiguity aversion if and only if care reduces the variance of their beliefs, that is, if and only if safety and reliability - as perceived by the victims - go hand in hand.

The impact of the “cautionary approach” underpinned by uncertainty aversion on the optimal standard is not univocal: the standard might increase (if the safest measures are the most reliable) or decrease (if the safest measures are the least reliable). A stronger dike may be prescribed (higher standard), if the impact of climate change on the sea level is uncertain and a stronger dike prevents harm under all circumstances. A new, more effective, therapeutic treatment may not be requested (lower standard),

⁷My characterization of full efficiency differs from that of Shavell (1982). I elaborate on this in Appendix A3.
⁸This case is equivalent to a regulatory regime in which the standard is set by an administrative agency and no compensation is awarded to the victims if the injurer has met the standard (under the so-called “regulatory compliance” defence).
if it entails uncertain teratogenic effects.\footnote{Since new prevention methods tend to have less certain effects than older ones - because less evidence is usually available - uncertainty aversion tends to introduce an “anti-innovation” attitude in safety policy. This is in line with the (survey) evidence collected by Viscusi (1999), who reports the tendency of judges to prefer old drugs with known effects to new, potentially more effective drugs, whose performance is less certain.}

In Section 5, I address the classic issue in liability design, and compare strict liability and negligence. Strict liability dominates negligence when the injurer is less averse to uncertainty (either because of lower degrees of risk and ambiguity aversion or because she is able to formulate more precise estimates of the probability of harm). The reverse does not hold true: if courts can award under-compensatory damages, strict liability may dominate negligence even if the victim is less averse to uncertainty than the injurer. Negligence definitely dominates strict liability, however, if the degrees of risk and ambiguity aversion of the victim are sufficiently small.

In Section 6, I considers some important extensions of the basic model. First, I show how the results can be adapted to the case in which also the victim can take precautions (“bilateral care”). I compare two liability rules: i) strict liability with the defence of contributory negligence, and ii) simple negligence. With minor qualifications, the main results apply to this setting too. Second, I discuss the case in which parties can purchase insurance from an insurer that is, just like real life insurance companies, uncertainty averse. Because of this, insurance contracts will contain a deductible and insured parties will bear some uncertainty. So uncertainty attitudes matter and the results of the previous sections apply again. Third, I consider the case in which harm is dispersed on many victims. Here, negligence tends to outperform strict liability, since it allows for better uncertainty spreading. Under strict liability, uncertainty spreading can only be obtained by setting damages to a very low level. If this is the case, however, the injurer loses her incentives to take care. Finally, I extend the analysis to the case in which ambiguity aversion leads the parties to formulate biased beliefs (“neo-additive model”). Parties thus suffer form an optimism or a pessimism bias. I show that, in this case, strict liability dominates negligence if the injurer is more confident in his belief, less pessimist, and less risk averse than the victim. When harms are dispersed and the
injurer is averse to risk, negligence dominates if the number of victims is sufficiently large.

Literature review

In his classic analysis of liability design under risk aversion, Shavell (1982) shows that strict liability is preferable when the injurer is risk-neutral and the victim risk-averse, while negligence is preferable in the opposite case. Beyond these polar cases, no further conclusion can be reached because attitudes towards risk themselves depend on the liability rule adopted.\(^\text{10}\) Another important result, found in Shavell (1982), is that optimal damages under strict liability are under-compensatory, in line with the basic insight of Mossin (1968). This result holds also in my (smooth) uncertainty model, as far as parties’ beliefs share the same mean.

The impact of ambiguity aversion on liability law is the focus of the pioneering contribution of Teitelbaum (2007), which relies on the neo-additive ambiguity model. As mentioned above, the neo-additive model introduces an optimism/pessimism bias in parties’ beliefs. Teitelbaum assumes that the court and the victim are uncertainty neutral, and investigates the impact of the injurer’s ambiguity aversion on her choice of care, which tends to be distorted. He shows that negligence tends to outperform strict liability, since under the latter the injurer has to make a finer decision (what level of care to take, rather than whether to meet the standard or not). In this sense, negligence allows for better “insulation” of non-rational behavior.\(^\text{11}\)

In my model, the uncertainty regarding the probability of harm is ingrained in the risky activity: it cannot be resolved. As such, it affects both the injurer and the victim.

\(^{10}\) Other investigations of optimal liability design under risk aversion include Greenwood and Ingene (1978), deriving optimal risk sharing rules using a local approach, and Graff Zivin and Small (2003), dealing with bilateral accidents with side payments under CRRA utilities. Nell and Richter (2003) address the case of dispersed harm under CARA utilities. Langlais (2010) investigates optimal risk allocation for perfectly correlated harms under RDEU. In Franzoni (2016), I investigate the relative desirability of strict liability and negligence when harms are correlated. Strict liability outperforms negligence when harms are negatively correlated. The opposite applies when harms are positively correlated.

\(^{11}\) See Jolls and Sunstein (2006) for a broad view on insulation and debiasing approaches in behavioural law and economics. Chakravarty and Kelsey (2017) extend Teitelbaum (2007) to bilateral accidents. In their model, care reduces the level of harm (rather than its probability). Parties formulate ambiguous beliefs about the level of care taken by the other side.
Furthermore, ambiguity aversion is taken as a *rational* response to scientific uncertainty, rather than a cognitive bias. As originally noted by Ellsberg (1961), ambiguity aversion is not a mistake that agents would be willing to correct once noted. Instead, ambiguity aversion is the manifestation of a rational doubt about the reliability of causal models.\(^{12}\)

If one takes this approach and includes ambiguity costs in welfare evaluations, then the relevant question becomes not how to force people to behave *as if* they were ambiguity-neutral, but rather how to design liability law so as to minimize their uncertainty burden.\(^{13}\)

## 2 A simple example: Hand’s rule

In order to appreciate the impact of uncertainty aversion on the optimal standard of care, let us consider a simplified case.

Suppose that an activity causes harm equal to 100 with probability 30%.

<table>
<thead>
<tr>
<th>harm</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>30%</td>
</tr>
<tr>
<td>0</td>
<td>70%</td>
</tr>
</tbody>
</table>

Let us calculate the cost of this prospect for the victim. Under uncertainty neutrality, the cost of this prospect boils down to the expected loss, which is equal to \(\bar{h} = 30\% \times 100 = 30\).

If the victim is *averse to risk*, he will also care about the variability of his payoff brought about by the accident. So, the cost of the harm prospect will depend negatively on the variance of the loss, which is:

\[
\sigma^2_h = \frac{30}{100} (100 - 30)^2 + \frac{70}{100} (0 - 30)^2 = 2100.
\]

The cost of the harm prospect for the risk-averse victim is, using a standard Arrow-Pratt

\(^{12}\)See the convincing arguments of Gilboa and Marinacci (2013), and references therein.

\(^{13}\)My approach to efficiency has two important positive collateral advantages. First, it is immune to Coasian bargaining: agents are not interested in modifying the efficient outcome by direct negotiations or other market arrangements. Second, the enforcement of efficient rules does not depend on unbiased, technocratic courts. This non-paternalistic approach is also taken by the theoretical literature on efficient ambiguity sharing. See, for instance, Strzalecki and Werner (2011) and literature cited therein.
approximation:

\[
Cost\ of\ harm\ prospect\ (risk) = \bar{h} + \frac{1}{2} \rho \sigma_h^2 = 30 + 1050 \rho,
\]

where \( \rho \geq 0 \) is an index that captures the degree of risk aversion of the victim. If \( \rho = 0.005 \) (a realistic estimate), the cost of the prospect is equal to 35.25, where 5.25 is the cost of uncertainty (risk premium).

Let us consider now the case in which the probability of harm is not known with certainty: for some experts it is equal to 10%, while for others it is 50%. Both estimates are equally plausible, so the victim attaches an equal weight to them. The prospect is now:

<table>
<thead>
<tr>
<th>harm</th>
<th>causal model</th>
<th>exp. loss</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>model A</td>
<td>100</td>
<td>10%</td>
<td>1/2</td>
</tr>
<tr>
<td>0</td>
<td>90%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>model B</td>
<td>100</td>
<td>50%</td>
<td>1/2</td>
</tr>
<tr>
<td>0</td>
<td>50%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1

Note that the expected probability of harm (from the two models) is still: \( \frac{1}{2} \times 10\% + \frac{1}{2} \times 50\% = 30\% \). For an ambiguity-neutral victim, this is all that matters. So, the cost of the new prospect is the same as before (cost of harm with risk aversion).

If the victim is averse to ambiguity, instead, the new prospect is worse than the previous one. The fact that expert opinions are conflicting leads the victim to cast a shadow of doubt on them, especially on the most optimistic one. The loss of welfare caused by scientific uncertainty is larger if the two opinions are further apart. To calculate the cost of the new prospect, one should take into account the variance of the expected loss across models: 

\[
\sigma^2 = \frac{1}{2} (10 - 30)^2 + \frac{1}{2} (50 - 30)^2 = 400.
\]

In the Appendix, I use a second order approximation to show that, for small losses, the cost
of harm takes the following handy shape:

Cost of harm prospect (risk and ambiguity) = \( \bar{h} + \frac{1}{2} \rho \sigma_h^2 + \frac{1}{2} \theta \sigma_\mu^2 = 30 + 1050 \rho + 200 \theta \),

where \( \theta \geq 0 \) is a parameter capturing the victim’s degree of ambiguity aversion.

Let us consider the case in which the aversion to ambiguity of the victim is extreme. In this case, the victim focusses only on the most adverse model (model B), as it were true for sure. In this example, this corresponds to an index of ambiguity aversion \( \theta = 0.1 \). The cost of the prospect for the victim is therefore \( 30 + 1050 \times 0.005 + 200 \times 0.1 = 56.5 \). The uncertainty premium is now equal to 26.5, where 5.25 is the risk premium and 21.25 the ambiguity premium.

Let us see how the previous observations affect Judge Hand’s rule. If a precautionary measure of cost \( c_0 \) could bring the probability of harm down to nil for sure, then the injurer should be deemed negligent for not taking such measure if, and only if:

\[
\text{Hand’s rule 1:} \quad c_0 < \bar{h} + \frac{1}{2} \rho \sigma_h^2 + \frac{1}{2} \theta \sigma_\mu^2.
\]

If, for example, \( \rho = 0.005 \) and \( \theta = 0.1 \), we get \( c_0 < 56.5 \) (instead of \( c_0 < 30 \) that would apply under uncertainty neutrality). Here, uncertainty aversion raises the standard of care: the lack of precautions generates costly uncertainty, so precautions become more valuable.

Let us consider now the case in which uncertainty is generated by the prevention measure itself. Let us suppose that the accident occurs with probability one for sure if no measures are taken. If a measure of cost \( c_1 \) is taken, then the probability of harm is that described in the harm prospect above (Table 1). Here, the injurer should be deemed negligent if: \( c_1 + \bar{h} + \frac{1}{2} \rho \sigma_h^2 + \frac{1}{2} \theta \sigma_\mu^2 < 100 \), that is

\[
\text{Hand’s rule 2:} \quad c_1 < 100 - \bar{h} - \frac{1}{2} \rho \sigma^2 (h) - \frac{1}{2} \theta \sigma_\mu^2.
\]

\(^{14}\)If Model B were true, then cost of the prospect would be equal to: 50 + 1250 \( \rho = 56.5 \). This upper limit is reached when \( \theta = 0.1 \).
If $\rho = 0.005$ and $\theta = 0.1$, we get: $c_1 < 100 - 56.5 = 43.5$ (instead of $c_1 < 70$ that would apply under uncertainty neutrality). Here, uncertainty aversion reduces the standard of care: since the impact of the precautionary measure is highly uncertain, it becomes less valuable.

These two basis examples illustrate the impact of uncertainty aversion on the optimal standard of care. The standard increases or decreases depending on the relationship between expected safety (the mean probability of harm) and uncertainty. In the first example, uncertainty decreases as the mean probability of harm drops from 30% to 0. In the second example, uncertainty increases as the mean probability of harm drops from 1 to 30%. More generally, the optimal standard increases if, and only if, precautionary measures reduce overall uncertainty.

If we look at the components of the uncertainty premium, we discover the following. For probabilities of harm reasonably small (less than $1/2$), the risk premium decreases with safety. So, risk aversion militates in favor an increase in the standard of care. The ambiguity premium, instead, may increase or decrease with safety. Additional precautions may decrease the ambiguity premium (if these measures are “reliable”) or increase it (if they are “not reliable”). If safety and reliability go hand in hand, ambiguity aversion reinforces the impact of risk aversion. If not, ambiguity aversion and risk aversion pull the optimal standard in opposite directions.

The following sections formalize the previous insights.

3 Uncertainty aversion

Let us consider the case in which an injurer (I) can cause harm $h$ to a victim (V). Assume first that the probability of harm is $\pi (x)$, where $x$ is the level of care taken by the injurer and $\pi (x)$ is common knowledge. Liability law determines whether the victim is entitled to recover damages from the injurer. Let us assume, for the time being, that damages $d$ are awarded. The victim bears loss $h - d$ in case of accident.
The expected utility of the victim, given the probability of harm $\pi (x)$, is

$$EU_{\pi(x)} = (1 - \pi (x)) u (i_V) + \pi (x) u (i_V - (h - d)),$$

where $i_V$ is his income.

If we take a second order approximation of the utility function, we obtain an expression for the payoff of the victim that depends only on the mean and the variance of the accident risk. The certainty equivalent $c_{\pi(x)}$ of this risk (such that $u (c_{\pi(x)}) = EU_{\pi(x)}$) can be written as:

$$c_{\pi(x)} = i_V - \pi (x) (h - d) - \frac{1}{2} \rho_V \sigma_{\pi(x)}^2 (h - d)^2,$$

where $\pi (x) (h - d)$ is the expected loss, $\sigma_{\pi(x)}^2 = \pi (x) (1 - \pi (x))$ the variance of the unit loss, and $\rho_V$ the Arrow-Pratt degree of absolute risk aversion of the victim (see Pratt (1964)).

Suppose now that the probability of harm is uncertain: the victim formulates beliefs about $\pi (x)$. Let $\mu_V$ be the probability distribution over beliefs $\pi (x)$. The more dispersed the beliefs, and the higher the ambiguity.

Figure 1 illustrates the case in which two (equally plausible) models of individual dose-response to an environmental chemical are available. Here, $x$ represents the level of care, i.e. the reduction in exposure, and $\pi (x)$ the probability of a harmful effect. Under model A the “safety threshold” is $x^A$, while under model B it is $x^B$.\footnote{See National Research Council (2009) for the extensive implications of scientific uncertainty, including dose-response, on risk assessment.}
For intermediate levels of care, ambiguity is high. Given the level of care \( x_0 \), for instance, the probability of harm is \( \pi_A \) under the first model and \( \pi_B \) under the second model. Under Expected Utility theory, the distribution of the beliefs is irrelevant: the only thing that matters is the mean probability of harm: \( p(x) = E_{\mu}(\pi(x)) \). This corresponds to the case of an ambiguity-neutral agent. Ambiguity-averse agents, instead, tend to dislike choice environments where probabilities are not known for sure (they are averse to mean preserving spreads of the beliefs). Instead of maximizing the simple mean of the Expected Utilities associated to the different beliefs, they maximize the mean of a concave transformation \( \varphi \) of the expected utilities. Fig. 2 illustrates.

\footnote{In this example, the relationship between care and ambiguity (dispersion of beliefs) is non-monotonic. In what follows, I make no specific assumptions on the shape of this relationship, but some results only apply if ambiguity decreases with care.}
Given $x_0$, under model A, the agent gets Expected Utility $EU(\pi_A)$, while under model B, he gets $EU(\pi_B)$. Aversion to ambiguity is captured by the curvature of the $\varphi$ function: the welfare the agent obtains from this ambiguous situation, $E_\mu(\varphi(EU(\pi)))$, is less than the welfare he would get if the probability of harm were $p(x)$ for sure, $\varphi(EU(p(x)))$.$^{17}$

By applying a second order approximation to the transformation function $\varphi$, I obtain an original mean-variance model (see Appendix A1), in which the certainty equivalent for the victim can be written as

$$C_V(x) = i_V - p(x)(h - d) - \frac{1}{2}\Psi_V(x)(h - d)^2. \quad (2)$$

$^{17}$Note how the smooth model (differently from most non-EU models) allows for a clear distinction between risk aversion (the curvature of the utility function), ambiguity (the dispersion of the beliefs), and ambiguity aversion (the curvature of the transformation function). If the agent is infinitely averse to ambiguity, he behaves as if the probability of harm were the largest among the possible ones (in his beliefs set), in line with the Maximin Expected Utility of Gilboa and Schmeidler (1989).
The uncertainty premium has a simple shape:

\[
\frac{1}{2} \Psi_V (x) (h - d)^2 = \frac{1}{2} \rho_V \sigma^2_{p(x)} (h - d)^2 + \frac{1}{2} \theta_V \sigma^2_{\mu_V} (\pi(x)) (h - d)^2. \tag{3}
\]

The uncertainty premium is equal to the sum of the risk and ambiguity premia. The **risk premium** is one half of the degree of risk aversion \( \rho_V \) times the variance of the loss generated by the mean probability of harm. The **ambiguity premium** is one half of the agent’s degree of ambiguity aversion \( \theta_V \) times the variance of expected loss generated by the prior beliefs. The uncertainty index:

\[
\Psi_V (x) = \rho_V \sigma^2_{p(x)} + \theta_V \sigma^2_{\mu_V} (\pi(x)) \tag{4}
\]

is thus a local measure of the costs of risk and ambiguity for the victim (for a unit loss). It depends on two taste parameters (\( \rho_V \) and \( \theta_V \)) and two uncertainty measures (\( \sigma^2_{p(x)} \) and \( \sigma^2_{\mu_V} (\pi(x)) \)).

In the following, I will assume that the victim (as the injurer) is neither risk nor ambiguity lover: \( \rho_V \geq 0, \theta_V \geq 0 \).\(^{18}\) Results can be easily adapted to the opposite case.

Before turning to the injurer, I need to make some assumptions on the structure of the beliefs. The injurer and the victim can have different priors about the probability of harm \( \pi(x) \). I assume, however, that these priors share the same mean \( p(x) \): parties formulate unbiased estimates of the probability of harm. In this sense, harm is “foreseeable.”\(^{19}\)

\(^{18}\)From auto collision insurance choices of households, the following baseline estimates of the absolute risk aversion index have been obtained: \( \rho \in [0.002, 0.008] \) (Barseghyan et al. (2013)), \( \rho \approx 0.0067 \) (Cohen and Einav (2007)), and \( \rho \in [0.002, 0.0053] \) (Sydnor (2010)). Several estimates of the smooth model are available. Conte and Hey (2013) get: \( \theta \approx 1.37 \) for a potential gain of 40 Euros. In Franzoni (2016) I review the reasons why firms tend to display risk aversion.

\(^{19}\)Foreseeability of risks is a common requisite for liability to apply (see Restatement (Third) of Torts: Liability for Physical Harm §3, 2010). This paper does not address scientifically unknowable risks. Additionally, because the information set does not chance with time, it does not consider hindsight liability. On the latter, see Ben-Shahar (1998),
Assumption 1  Foreseeability. For all levels of care $x \geq 0$, the beliefs of the injurer and the victim share the same mean: $E_{\mu_I}(\pi(x)) = E_{\mu_V}(\pi(x)) \equiv p(x)$.

Parties do not fundamentally disagree on the probability of harm. They might however estimate these probabilities with different degrees of precision (extent of ambiguity). As the ambiguity entailed in the beliefs drops to zero, the model converges to the standard EU model, in which $p(x)$ is regarded as the “true” probability of harm.\footnote{20}{To rationalize: when the evidence on harm is scarce or is conflicting, beliefs will be ambiguous. Here, $p(x)$ is just the mean belief. As the evidence accumulates, ambiguity disappears and $p(x)$ emerges as the “objective” probability of harm, known to all parties.}

Care is assumed to reduce the mean probability of harm at a decreasing rate. To better disentangle the effect of risk aversion, I further assume that the mean probability of harm is not too large.

Assumption 2  For any level of care: $p(x) \leq 1/2$, $p'(x) < 0$, and $p''(x) > 0$.

The variance of the unit loss generated by the mean probability of harm is $\sigma^2_{p(x)} = p(x)(1 - p(x))$. This variance decreases with $x$ if Assumption 2 is met.\footnote{21}{Assumption 2 is stronger than necessary. What I really need is that the mean probability of harm is not greater than 1/2 in equilibrium. Conducts yielding an equilibrium probability of harm greater than 1/2 would probably qualify as reckless and wanton. The results of the paper can be easily adapted to case in which $p(x) > 1/2$. In this case, risk aversion militates in favor of a decrease in the standard of care.}

Let us now consider the injurer, on the assumption that she pays damages $d$ when an accident occurs. Her certainty equivalent can be written as:

$$C_I(x) = i_I - x - p(x)d - \frac{1}{2}\Psi_I(x)d^2,$$

(5)

where $i_I$ is her income, $x$ the expenditure in care, and $\Psi_I(x)$ her uncertainty index:

$$\Psi_I(x) = \rho_I\sigma^2_{p(x)} + \theta_I\sigma^2_{\mu_I}(\pi_J(x)).$$

(6)

Again, the uncertainty index includes a risk aversion component (index of risk aversion $\rho_I$ times the variance of the mean probability of harm) and an ambiguity aversion component (index of ambiguity aversion $\theta_I$ times the variance of her beliefs).
As the injurer invests more in care $x$, the expected liability payment $p(x)d$ decreases, the risk premium $\frac{1}{2}\rho_I \sigma_{p(x)}^2 d^2$ decreases, while the ambiguity premium can increase or decrease, depending on whether care increases or decreases the uncertainty regarding the probability of harm.

4 Optimal liability design

4.1 Strict liability

Under strict liability, the injurer pays compensation to the victim irrespective of the amount $x$ invested in precaution. Courts can affect the injurer’s behavior and the allocation of uncertainty by means of the damages awarded $d$. Damages can fully compensate the victim ($d = h$), they can overcompensate him, e.g., by including a punitive component, or they can under-compensate him, e.g., when caps are imposed or when some types of harm are deliberately excluded (e.g., pain and suffering).

The optimal level of $d$ is obtained from the maximization of Social Welfare

$$SW^{SL} = C_I(x_0) + C_V(x_0)$$

$$= i_I - x_0 - p(x_0) d - \frac{1}{2}\Psi_I(x_0) d^2 + i_V - p(x_0)(h - d) - \frac{1}{2}\Psi_I(x_0)(h - d)^2.$$ 

Since $i_I$ and $i_V$ are constants, the problem is equivalent to the minimization of Social Loss:

$$\min L^{SL}(d) = x_0 + p(x_0) h + \frac{1}{2}\Psi_I(x_0) d^2 + \frac{1}{2}\Psi_V(x_0)(h - d)^2.$$ 

(7)

The care level $x_0$ is chosen by the injurer so as to maximize her certainty equivalent $C_I(x)$. She will set $x_0$ so that

$$1 = - p'(x_0) d - \frac{1}{2}\Psi_I'(x_0) d^2 :$$

(8)

---

22Since the certainty equivalents are monotone transformations of the welfare levels of the parties, the maximization of their sum yields an ex-ante Pareto efficient outcome. Because certainty equivalents are independent of income, the efficient outcome is unique (up to direct income transfers).

23Here and below, I assume that the mean probability of harm $p(x)$ is convex enough to make the maximization problems quasi-concave.
an additional dollar spent in precaution reduces her expected liability and her uncertainty premium by one dollar.

We have:

\[ \Psi' \left( \frac{\partial \sigma_I(x)}{\partial x} \right) = \rho \frac{\partial \sigma_I(x)}{\partial x} (1 - 2p(x)) + \theta \frac{\partial \sigma_I(x)}{\partial x}. \]  

(9)

An increase in care reduces the variance of the mean probability of harm \( p(x) \) (thanks to Assumption 2) and affects the level of ambiguity borne by the injurer. If care does not increase the variance of her prior, \( \frac{\partial \sigma_I(x)}{\partial x} \leq 0 \), then: \( \Psi' (x) < 0 \).

From (9) and (8), we get that incentives to take care increase with the index of risk aversion \( \rho_I \), while they increase with the index of ambiguity aversion \( \theta_I \) if, and only if, \( \frac{\partial \sigma_I(x)}{\partial x} < 0 \). So, ambiguity aversion leads the injurer to take additional precautionary measures if, and only if, these measures are reliable (they reduce the uncertainty about prospective liability).

Care increases with damages \( d \) if, and only if, the following condition holds:

\[ p'(x_0) + \Psi' (x_0) d < 0, \]  

(10)

Condition (10) posits that care can increase ambiguity, but not too much. In what follows, I will assume that condition (10) is met for all levels of \( d \).

By differentiation of (7), using (8), we get:

\[ \frac{\partial L^{SL}(d)}{\partial d} = \frac{\partial x_0}{\partial d} \left[ p'(x_0) (h - d) + \frac{1}{2} \Psi_V (x_0) (h - d)^2 \right] + \Psi_I (x_0) d - \Psi_V (x_0) (h - d). \]  

(11)

An increase in damages has two effects: i) it provides the injurer with additional incentives to take care and hence to reduce the “externality” she exerts on the victim (uncompensated harm and the attendant uncertainty premium), and ii) it shifts the

\[ 24 \text{Eq. (10) can be written as } \theta \frac{\partial \sigma_I(x)}{\partial x} < -p' \left[ \frac{1}{d} + (1 - 2p) \rho_I \right]. \]  

It is met if \( -p' \) is sufficiently large (care reduces substantially the mean probability of harm), \( \theta_I \) is small, \( \rho_I \) is large and \( \frac{\partial \sigma_I(x)}{\partial x} \) is small. Admittedly, if \( \frac{\partial \sigma_I(x)}{\partial x} > 0 \) and the injurer is extremely averse to ambiguity, then eq. (10) may not hold.
uncertainty burden from the victim to the injurer.

By implicit differentiation of (11), we get: $\frac{\partial d^*}{\partial \rho_V} > 0$. Furthermore, if $\frac{\partial \sigma_{V}^2(x_0)}{\partial x} \leq 0$, then $\frac{\partial d^*}{\partial \theta_V} > 0$.

Note that for $d = h$, the “externality” effect vanishes and marginal social loss collapses to

$$\frac{\partial L^{SL}(h)}{\partial d} = \Psi_I(x_0) h \geq 0.$$ 

With fully compensatory damages ($d = h$), incentives to take care would be appropriately set, since the injurer fully internalizes the consequences of her actions. However, the uncertainty would not be optimally allocated, since all of the burden would be placed on the injurer. If the injurer is not uncertainty neutral (i.e., if $\Psi_I(x_0) > 0$), the allocation of uncertainty can be improved at the margin - with a negligible effect on the welfare of the victim - by reducing damages and shifting some of the uncertainty on the victim. The benefit for the injurer is of the first order, the cost for the victim of the second order.\(^{25}\)

**Proposition 1** *Strict liability. If the injurer is not uncertainty neutral, damages should be less than fully compensatory: $d^* < h$. Optimal damages increase with the victim’s index of risk aversion $\rho_V$. They increase with the victim’s index of ambiguity aversion $\theta_V$, if care reduces the variance of the victim’s prior beliefs.*

Ambiguity aversion prescribes higher damages if the safest prevention measures are also the most reliable ones, from the victim’s point of view.

### 4.2 Negligence

Under a negligence rule, the injurer pays compensatory damages $d = h$ only if she does not meet the due standard of care $\bar{x}$. Care is assumed to be verifiable in court. Unless the standard is prohibitively high, the injurer will prefer to meet it and avoid liability.

\(^{25}\)This is a local result, so it extends beyond the mean-variance model. Note, further, that in this model the Injurer cannot escape responsibility, so the standard rationale for punitive damages does not apply.
I will thus assume that $x = \bar{x}$.\footnote{The injurer prefers to be negligent if $\bar{x} > x^* + p(x^*)h + \frac{1}{2}\Psi_I(x^*)h^2$, where $x^*$ maximises the injurer’s welfare when she is liable. If that is the case, however, then $\bar{x}+p(\bar{x})h +\Psi_V(\bar{x})h^2 > x^*+ p(x^*)h + \frac{1}{2}\Psi_I(x^*)h^2$, and the lawmaker itself would definitely prefer that the injurer did not meet the standard. The results of this paper also hold if damages (paid by the negligent injurer) differ from harm. However, damages cannot be too low, as otherwise the injurer prefers not to meet the standard and strict liability de facto applies.} The optimal standard should be set so as to minimize

$$L^N(\bar{x}) = \bar{x} + p(\bar{x})h + \frac{1}{2}\Psi_V(\bar{x})h^2.$$\hspace{1cm} (12)

All the uncertainty is borne by the victim.

The optimal standard $\bar{x}$ should solve:

$$1 = -p'(\bar{x})h - \frac{1}{2}\Psi_V'(\bar{x})h^2 :$$ \hspace{1cm} (13)

an additional dollar spent on precaution should reduce expected harm and the uncertainty premium of the victim by one dollar.

We get $\frac{\partial\bar{x}}{\partial h} > 0$ if, and only if: $p'(\bar{x}) + \Psi_V'(\bar{x})h < 0$, which is assumed to hold.\footnote{This assumption is the equivalent of Condition (10) applied to the Injurer.}

From (4) and Assumption 2, by implicit differentiation, we get: $\frac{\partial\bar{x}}{\partial \rho_V} > 0$. Furthermore, $\frac{\partial\bar{x}}{\partial \theta_V} > 0$ if, and only if: $\frac{\partial \sigma^2_{\rho_V}(\pi(\bar{x}))}{\partial x} < 0$. Hence, the following.

**Proposition 2** The optimal standard of care increases with the victim’s degree of risk aversion $\rho_V$. It increases with the victim’s degree of ambiguity aversion $\theta_V$ if, and only if, care reduces the variance of the victim’s prior beliefs.

The latter result precisely identifies the impact of scientific uncertainty on the optimal standard. The standard increases if, and only if, safety and reliability - from the victim’s point of view - go hand in hand. The optimal standard should take into account both dimensions of prevention. If the safest prevention measures turn out to be the least reliable, then uncertainty aversion calls for a reduction in the standard.
5 Strict liability vs. negligence

Both negligence and strict liability provide second best solutions to the concomitant problems of optimal uncertainty allocation and harm prevention. Which rule is preferable? Under negligence, the uncertainty is fully placed on the victim and the standard of care is (optimally) set by the courts. Under strict liability, uncertainty is shared at the optimum, and the level of care is chosen by the injurer.

To compare strict liability and negligence, let us start from the special case in which damages are equal to harm: \( d = h \). Here, constrained social loss amounts to:

\[
\hat{L}^{SL}(x^c) = x^c + p(x^c)h + \frac{1}{2} \Psi_I(x^c)h^2, \tag{14}
\]

where \( x^c \) is the level of care chosen by the injurer (eq. 8).

Under negligence, we have instead

\[
L^N(x^n) = x^n + p(x^n)h + \frac{1}{2} \Psi_V(x^n)h^2, \tag{15}
\]

where \( x^n \) is optimally chosen by the courts.

If, under strict liability with compensatory damages, the injurer were forced to take the level of care \( x^n \), social loss would amount to

\[
\hat{L}^{SL}(x^n) = x^n + p(x^n)h + \frac{1}{2} \Psi_I(x^n)h^2, \tag{16}
\]

with

\[
\hat{L}^{SL}(x^n) < L^N(x^n) \iff \Psi_I(x^n) < \Psi_V(x^n). \]

Let us suppose that the latter inequality is met: shifting the loss from the victim to the injurer, keeping the level of care constant, reduces social loss. In fact, social loss would be even lower if the injurer were free to chose the level of care \( x^c \) that maximizes her welfare (under full compensation, the welfare level of the victims is not affected):

\[
\hat{L}^{SL}(x^c) \leq \hat{L}^{SL}(x^n). \]

Social loss would further decline if damages were optimally set by the courts:

\[
L^{SL}(x_0) < \hat{L}^{SL}(x^c), \]

where \( x_0 \) is the level of care taken by the injurer when \( d = d^* < h \).
To sum up, strict liability dominates negligence if the following Condition holds:

\[ \Psi_I(x^n) \leq \Psi_V(x^n). \]

**Condition I** identifies the Injurer as the best uncertainty bearer.

Let us consider the opposite case. Given the level of care \( x^c \) chosen by the injurer under strict liability with compensatory damages, shifting the loss on the victim reduces social loss if the following condition holds:

\[ \Psi_I(x^c) > \Psi_V(x^c). \]

**Condition V** identifies the Victim as the best risk bearer. If Condition V holds, then negligence with \( \bar{x} = x^c \) dominates strict liability with compensatory damages. Social loss would further decline if, under negligence, the level of care were optimally chosen by the courts: \( L^N(x^n) \leq L^N(x^c) \).

We have thus proved the following.

**Proposition 3** *Strict liability vs. negligence.*

i) if Condition I holds, strict liability dominates negligence.

ii) If Condition V holds, negligence dominates strict liability with compensatory damages \((d = h)\).

Proposition 3 identifies some clear-cut conditions for the choice between liability rules. These conditions are based on the ability of the parties to tolerate uncertainty. In turn, this ability depends on their degrees of aversion to risk and ambiguity, and on the dispersion of their beliefs.\(^{28}\)

The result, however, is not symmetrical: when courts can suitably chose the level of damages, Condition V is not enough to guarantee the dominance of negligence. The latter can only be obtained under stronger conditions.

From (12) and (7), one can see that

\[ L^N(x^s) < L^S(x^s) \iff \Psi_V(x^s) h^2 < \Psi_I(x^s) d^* + \Psi_V(x^s) (h - d^*)^2, \]

\(^{28}\)If \( \sigma^2_V(\pi(x)) \) and \( \sigma^2_I(\pi(x)) \) are affected in different ways by \( x \), one can have situations in which neither Condition I nor V hold (the victim might be the best uncertainty bearer for \( x^n \) but not for \( x^c \)).
where \( x^s \) is the care level chosen by the injurer under strict liability, which simplifies to

\[
L_N(x^s) < L_S(x^s) \iff \frac{d^*}{h} > 2 \frac{\Psi_V(x^s)}{\Psi_V(x^s) + \Psi_I(x^s)}.
\] (17)

Since \( d^* < h \), inequality (17) can be met only if \( \Psi_V(x^s) < \Psi_I(x^s) \).

Note that \( \frac{\Psi_V(x^s)}{\Psi_V(x^s) + \Psi_I(x^s)} \) converges to zero if \( \Psi_V(x^s) \) becomes small, while optimal damages \( d^* \) do not (the injurer needs to be incentivized, see Appendix A3). Thus, inequality (17) is met if \( \rho_V \) and \( \theta_V \) are both sufficiently small. If this is the case, then

\[
L_N(x^n) \leq L_N(x^s) < L_S(x^s),
\]

where \( x^n \) is the optimal level of care chosen by the courts: negligence dominates strict liability.

**Proposition 4** When damages are optimally set by the courts, negligence dominates strict liability if the uncertainty index of the victim is sufficiently small.

Also for this Proposition, note that the condition used is sufficient but not necessary.

Since both liability rules are able to provides incentives to take precautions (under mild assumptions), the comparison between them hinges on the way in which they allocate uncertainty: fully on the victims (negligence) vs. mostly on the injurer (strict liability). Strict liability is marginally superior, as it allows for uncertainty sharing.

### 6 Extensions

This section considers several important extensions of the basic model.

**Bilateral accidents**

In many cases, victims can take measures that reduce the likelihood of harm. If they fail to take them, they may be deemed negligent and may fail to recover damages. I compare two liability rules: i) strict liability with the defence of contributory negligence, and ii) simple negligence. Under the first rule, damages are awarded if, and only if, the
victim has taken due care. Under the latter rule, damages are awarded if, and only if, the injurer has not taken due care.\textsuperscript{29}

Both liability rules are able to provide incentives to take care.\textsuperscript{30} Under strict liability with contributory negligence, the injurer selects care so as to minimize the difference between his liability burden and the cost of care; the victim takes care so as to avoid liability. Optimal damages are under-compensatory. Under negligence, the injurer meets the standard of care; the victim selects care so as to minimize the difference between the burden of harm and the cost of care. The optimal standard of care for the injurer increases with the victim’s degree of risk aversion. It increases with the victim’s degree of ambiguity aversion if care (of both sides) reduces the variance of the victim’s prior (on the assumption that cross-effects do not go in the opposite direction or that they are small enough).

In the comparison between strict liability and negligence, Propositions 3 and 4 apply.

**Insurance**

If parties could purchase insurance at an actuarially fair price, then uncertainty aversion would become irrelevant: the loss bearing party would purchase full insurance and act as if it were uncertainty neutral. Unfortunately, this solution is generally unavailable: insurance companies do not charge actuarially fair prices. The reasons for this are twofold. First, the management and the organization of insurance companies give rise to substantial administrative costs, in the order of 30% of the premium (see Harrington and Niehaus (2003)). Second, insurance companies need to meet solvency requirements that induce them to behave like uncertainty averse agents (see Baker and Siegelman (2013)). Aversion both to risk and ambiguity ("parameter uncertainty") has been documented (see Kunreuther et al. (1995) and references therein).

If we extend the model and allow parties to purchase insurance from an uncertainty-averse insurance company, then the results of the previous sections apply again (see Supplementary material S2). Since optimal insurance contracts contain deductibles, parties bear uncertainty. Thus, their disposition towards uncertainty remains central.

\textsuperscript{29}Other rules are available, like comparative negligence and negligence with the defence of contributory negligence. They all essentially produce the same outcome. See Shavell (2007).

\textsuperscript{30}The analysis is provided in Supplementary material S1.
and Propositions 3 and 4 apply again.

**Dispersed harm**

Let us consider the case in which victims are numerous. For simplicity, I posit that all $n$ victims are alike: they all suffer harm $h$, and they are all equally averse to uncertainty: for all $j = \{1, 2, ..., n\}$ and for all $x \geq 0$, $\Psi_{Vj}(x) = \Psi_V(x)$.

Negligence dominates strict liability with fully compensatory damages if:

$$x^s + p(x^c) nh + \frac{1}{2} \Psi_I(x^c)(nh)^2 > x^s + np(x^c) h + n \frac{1}{2} \Psi_V(x^c) h^2 \iff \Psi_I(x^c) > \frac{\Psi_V}{n}(x^c).$$

For $n$ sufficiently large, ineq. (18) is surely met. So, if the number of victims is sufficiently large, negligence dominates strict liability.

The intuition for this result is straightforward. Since the Uncertainty Premium increases more than proportionally with the loss, the latter should not be entirely placed on the injurer. A liability rule spreading uncertainty tends to be preferable, as originally argued by Nell and Richter (2003) with respect to standard risk. Negligence can serve this goal, while strict liability cannot. If, under strict liability, damages were set at a very low level (so as to leave the largest share of the loss on the victims), the injurer would have too little incentive to take care.

**Pessimism/optimism.** A substantial departure from EU theory obtains if parties do not agree on the expected probability of harm: $p_I(x) \neq p_V(x)$. This is the case, for example, in Teitelbaum (2007)'s model, which builds on the new-additive ambiguity model. In his model, parties “distort” the true probability of harm because they lack confidence in it. The more severe this lack of confidence, the further away they move from Expected Utility by attaching greater weight to extreme payoffs (minimum and

---

31Note that also Proposition 3, part ii) applies: negligence dominates strict liability with optimal damages. Just replace $\Psi_V/n$ for $\Psi_V$ in the proof of Proposition 3.

32Nell and Richter (2003) prove that, for $n \to \infty$, first best damages collapse to nil (see Appendix A4). So, negligence is approximately first best efficient (if care is bounded above). All results of this paper, instead, refer to second best efficiency.
Thus, agents tend to display either pessimism (for low probability events) or optimism (for high probability events).

Since the neo-additive model is formally equivalent to the well known RDEU model (and Cumulative Prospect Theory, under symmetric treatment of gains and losses), in what follows I highlight several interesting implications of this approach (see Supplementary material S3 for an extensive analysis).

First, let us consider optimal standard setting under negligence. If victims have little confidence in the probability of harm, they will not fully savor the benefits of greater care. In the extreme case of complete lack of confidence, victims will regard all “possible” outcomes in the same way (50% likely), independently of the level of care taken by the injurer. The consequence of “likelihood insensitivity” is that the efficient standard will be very low (marginal costs are savored, marginal benefits are not).

Second. Let us consider optimal uncertainty sharing under strict liability. When parties are pessimist (they attach large weight to minimum utility), very small losses can have a substantial impacts on their welfare. Given compensatory damages, a small reduction of $d$ has a first-order effect on the welfare level of the victim, so it might be undesirable (while it has a second order effect under standard risk aversion and smooth ambiguity aversion). In particular, if the victim is substantially more pessimist than the injurer, compensatory damages are optimal ($d^* = h$).\textsuperscript{33}

Finally, if we compare strict liability and negligence, we get that strict liability dominates negligence if the injurer is more confident in his belief, less pessimist, and less risk averse than the victim. Interestingly, if harm is dispersed and the injurer is risk averse, negligence dominates whatever the degree of pessimism of the victims.

7 Final remarks

Scientific uncertainty looms large in the policy debate on toxic harms, where model uncertainty dwarfs standard stochastic risk (see Breyer (2009)). Given that uncertainty

\textsuperscript{33}Neo-additive ambiguity aversion elevates uncertainty to a first-order effect, as shown, in a more general set-up, by Lang (2017). Ahn et al. (2014) provide evidence of first-order ambiguity aversion in an experimental setting.
aversion is an established fact, liability law should come to terms with it. On the basis of a second-order approximation, this article has tried to shed light on the impact of standard risk and scientific uncertainty of optimal liability law. The lessons learned have a bearing on safety policy in general.

Uncertainty aversion introduces a new dimension for the evaluation of prevention measures: their impact on the dispersion of the beliefs (“reliability”). As noted above, precautionary measures can increase or decrease the dispersion of the beliefs: for this reason the optimal standard of care (under negligence) might go up or down. This observation implies that a cautionary policy is not necessarily one in which additional precautionary measures are taken: when these measures have uncertain effects, caution advises against them.

In the comparison between liability rules, what ultimately matters is the ability of the parties to bear uncertainty. In turn, this ability depends on their indexes of risk and ambiguity aversion, and on the variance of their beliefs. These are variables that can be empirically estimated. For this purpose, the extensive empirical results of the literature of risk perception provide a solid starting point (see Slovic (2010) for an introduction).

At a first cut, one might conjecture that, for toxic harms, injurers/firms can formulate more accurate estimates of the degree of safety of their activities. They are also likely to have smaller degrees of uncertainty aversion (possibly because they can diversify risks). So, in cases in which the number of (potential) victims is limited, strict liability is likely to induce a better allocation of uncertainty. This is in line with current environmental law, which is largely based on strict liability.

When harm is distributed on many victims, however, the optimal choice tilts towards negligence. In this case, if the injurer is risk-averse, risk spreading takes priority, independently of the uncertainty aversion (and the pessimism) of the victims. In an uncertain environment, the dose ultimately makes the poison. When harms are dispersed, it would not be wise to place the whole uncertainty burden on the producers, unless they are public companies completely indifferent to risk.

As a final caveat, I would like to emphasize that my analysis ignores many factors pertinent to liability law that are relevant for the policy debate. Among these, the
administrative costs of the tort and insurance systems, the cost of litigation, and the overlap between liability law and federal regulation.
Appendix

A1. Mean-Variance approximation. Given the investment in care \( x \), let \( \mu \) be the probability distribution describing the agent’s beliefs about the probability of harm \( \pi(x) \). These probabilities can belong to an interval or a discrete set. For any probability \( \pi(x) \), the expected utility of the agent is:

\[
EU_{\pi(x)} = (1 - \pi(x)) u(i) + \pi(x) u(i - \ell),
\]

where \( i \) is her net income and \( \ell \) the loss. The welfare functional is

\[
W = E_{\mu}(\varphi(EU_{\pi(x)})),
\]

where \( E_{\mu} \) is the expectation over the prior distribution of \( \pi(x) \), and \( \varphi \) a function capturing the agent’s attitude towards ambiguity.

If \( \varphi \) is linear, the maximization of \( W \) is equivalent to the maximization of

\[
\overline{W} = E_{\mu}(EU_{\pi(x)}) = (1 - E_{\mu}(\pi(x))) u(i) + E_{\mu}(\pi(x)) u(i - \ell) = EU_{p(x)},
\]

where \( p(x) = E_{\mu}(\pi(x)) \) is the mean probability of harm. In this case, the agent behaves like an Expected Utility maximizer: she only cares about the reduced probability \( p(x) \) of the compound lottery, and is said to be “ambiguity neutral.” If \( \varphi \) is concave, the agent is averse to mean preserving spreads of the beliefs:

\[
E_{\mu}(\varphi(EU_{\pi(x)})) < \varphi(E_{\mu}(EU_{\pi(x)})).
\]

For any probability \( \pi(x) \), the certainty equivalent of the harm lottery (such that \( u(c_{\pi(x)}) = EU_{\pi(x)} \)) can be written as:

\[
c_{\pi(x)} = i - \pi(x) \ell - \frac{1}{2} \rho \sigma^2_{\pi(x)} \ell^2 + o(\ell^2),
\]

where \( \pi(x) \ell \) is the expected loss, \( \sigma^2_{\pi(x)} = \pi(x)(1 - \pi(x)) \) the variance of the unit loss, \( \rho \) the Arrow-Pratt degree of absolute risk aversion of the utility function, and \( o(\ell^2) \) an expression that includes terms of third and higher order. If the loss is small or if \( u''' \) is close to zero, the last term can be neglected.

If we let

\[
w_{\pi(x)} = \pi(x) + \frac{1}{2} \rho \sigma^2_{\pi(x)} \ell,
\]

29
the certainty equivalent becomes: \( c_{\pi(x)} \simeq i - \ell \) \( w_{\pi(x)} \), where \( w_{\pi(x)} \) is a random variable that depends on \( \pi(x) \).

Let \( v(i) = \varphi(u(i)) \). We can write the welfare functional as

\[
W = E_{\mu}(v(c_{\pi(x)})).
\]

By using a second order expansion, we get an approximation for the total certainty equivalent \( C(x) \), with \( v(C(x)) = E_{\mu}(v(c_{\pi(x)})) \), which takes into account the uncertainty over \( w_{\pi(x)} \):

\[
C(x) = i - \ell E_{\mu}\left(w_{\pi(x)}\right) - \frac{1}{2} \lambda_{v} \ell^{2} \sigma_{\mu}^{2}\left(w_{\pi(x)}\right) + o\left(\ell^{2}\right),
\]

where \( \lambda_{v} = -\frac{\nu''(i-\ell) E_{\mu}(w_{\pi(x)})}{\nu'(i-\ell) E_{\mu}(w_{\pi(x)})} \) is the Arrow-Pratt index of absolute risk aversion of the \( v \) function. From eq. (21), we get

\[
E_{\mu}(w_{\pi(x)}) = p(x) + \frac{1}{2} \rho \ell E_{\mu}\left(\sigma_{\pi(x)}^{2}\right),
\]

and

\[
\sigma_{\mu}^{2}(w_{\pi(x)}) = \sigma_{\pi}^{2}(\pi(x)) + \frac{1}{2} \rho \ell^{2} \sigma_{\pi}^{2}(\pi(x)) + 2Cov\left(\pi(x), \frac{1}{2} \rho \ell \sigma_{\pi}^{2}(\pi(x))\right).
\]

Thus (omitting the argument of \( \pi(x) \)),

\[
C(x) = i - \ell \left[ p(x) + \frac{1}{2} \rho \ell E_{\mu}\left(\sigma_{\pi}^{2}\right) \right] - \frac{1}{2} \lambda_{v} \ell^{2} \left[ \sigma_{\pi}^{2}(\pi) + \left(\frac{1}{2} \rho \ell\right)^{2} \sigma_{\pi}^{2}(\sigma_{\pi}^{2}) + \rho \ell Cov\left(\pi, \sigma_{\pi}^{2}\right) \right] + o\left(\ell^{2}\right) =
\]

\[
i - p(x) \ell - \frac{1}{2} \rho \ell^{2} E_{\mu}\left(\sigma_{\pi}^{2}\right) - \frac{1}{2} \lambda_{v} \ell^{2} \sigma_{\pi}^{2}(\pi) - \ell^{4} \left(\frac{1}{2} \rho \ell\right)^{2} \sigma_{\pi}^{2}(\sigma_{\pi}^{2}) - \frac{1}{2} \lambda_{v} \rho \ell^{3} Cov\left(\pi, \sigma_{\pi}^{2}\right) + o\left(\ell^{2}\right) =
\]

\[
i - p(x) \ell - \frac{1}{2} \rho \ell^{2} E_{\mu}\left(\sigma_{\pi}^{2}\right) - \frac{1}{2} \lambda_{v} \ell^{2} \sigma_{\pi}^{2}(\pi) + o\left(\ell^{2}\right).
\]

The total uncertainty equivalent is thus approximately equal to: i) income less the expected loss, ii) less the mean of the Arrow-Pratt risk premium, iii) less a term which depends on the variance of the belief.

Since \( v(i) = \varphi(u(i)) \) and \( \frac{\mu''}{\varphi'} = \frac{\nu''}{\varphi'} + \frac{\nu''}{\nu'}, \) we get \( \lambda_{v} = \theta + \rho, \) with \( \theta = -\frac{\nu''}{\varphi'} \). Hence, omitting

\[\text{Footnote 34:} \text{I am grateful to Fabio Maccheroni and Massimo Marinacci for providing the steps needed to reconcile my approximation to their general result (presented in Maccheroni et al. (2013), Appendix A1). My approximation applies to downside risks, theirs to symmetric risks. Related approximations are obtained by Jewitt and Mukerji (2011) and Izhakian and Benninga (2011).}\]
third and higher order terms:

\[ C(x) \simeq i - p(x) \ell - \frac{1}{2} \rho \ell^2 E_\mu \left( \sigma^2_{\pi(x)} \right) - \frac{1}{2} \left[ \theta + \rho \right] \ell^2 \sigma^2_{\mu} (\pi (x)) \]

\[ \simeq i - p(x) \ell - \frac{1}{2} \rho \ell^2 \left[ E_\mu \left( \sigma^2_{\pi(x)} \right) + \sigma^2_{\mu} (\pi (x)) \right] - \frac{1}{2} \theta \ell^2 \sigma^2_{\mu} (\pi (x)). \]

Note that

\[ E_\mu \left( \sigma^2_{\pi(x)} \right) = E_\mu (\pi (x) (1 - \pi (x))) \]

\[ = p(x) - E_\mu (\pi (x)^2) = p(x) - \left[ \sigma^2_{\mu} (\pi (x)) + p(x)^2 \right] \]

\[ = p(x) (1 - p(x)) - \sigma^2_{\mu} (\pi (x)) = \sigma^2_{\mu (x)} - \sigma^2_{\mu} (\pi (x)). \]

Thus,

\[ C(x) \simeq i - p(x) \ell - \frac{1}{2} \rho \sigma^2_{\mu (x)} \ell^2 - \frac{1}{2} \theta \sigma^2_{\mu} (\pi (x)) \ell^2. \]

Under ambiguity aversion, the certainty equivalent is equal to income less expected loss less the risk premium attendant with the mean probability \( p(x) \), less an ambiguity premium which depends on the variance of the beliefs \( \sigma^2_{\mu} (\pi (x)) \). The uncertainty premium \( UP(x) = E_\mu (\pi (x) \ell) - C(x) \) is therefore:

\[ UP(x) = \frac{1}{2} \rho \sigma^2_{\mu (x)} \ell^2 + \frac{1}{2} \theta \sigma^2_{\mu} (\pi (x)) \ell^2 \equiv \frac{1}{2} \Psi (x) \ell^2. \] (22)

The uncertainty premium is equal to the sum of the risk and ambiguity premia.

A2. Strict liability vs. negligence. We have \( L^N (x^*) < L^S (x^*) \) if, and only if (omitting arguments):

\[ \frac{d^*}{h} > \frac{2 \Psi_V}{\Psi_I + \Psi_V}. \] (23)

If the victim is risk and ambiguity neutral \( (\rho_V \to 0, \theta_V \to 0) \), we have, from eq. (11)

\[ L^{St} = \frac{\partial x^o}{\partial d} p' (h - d) + \left[ p (1 - p) \rho_I + \theta_I \sigma^2_{\mu I} \right]. \] (24)

For \( d \to 0 \), we get

\[ L^{St} (0) = \frac{\partial x^o}{\partial d} p' h < 0 ; \]

optimal damages \( d^* \) do not drop to zero when the victim is uncertainty neutral. Thus, \( \frac{d^*}{h} \) is
surely greater than $\frac{2 \Psi V}{\Psi I + \Psi V}$ if $\Psi V$ is sufficiently small. This proves Proposition 4.

**A3. Efficient loss sharing.** Let us consider the allocation that parties themselves would agree on if they could write an ex-ante contract specifying the level of precaution to be taken by the injurer and the way in which the loss is shared. This contract would also specify an up-front transfer between them.\(^{35}\) Let us consider the case with $n$ identical victims.

Full efficiency is obtained from the maximization of Social Welfare

$$SW = C_I + nC_V = i_I - x - p(x) nd - \frac{1}{2} \Psi_I(x) (nd)^2 + n [i_V - p(x)(h - d) - \frac{1}{2} \Psi_I(x) d^2],$$

Since $i_I$, $i_V$, and $h$ are constants, the problem is equivalent to the minimization of Social Loss:

$$\min L = x + p(x) nh + \frac{1}{2} \Psi_I(x) (nd)^2 + n\frac{1}{2} \Psi_V(x) (h - d)^2.$$  

Social loss includes the expenditure in prevention, expected harm, and the uncertainty premia associated with the risk of harm.

From the minimization of $L$, we get:

$$\frac{\partial L}{\partial x} = 1 + p'(x) nh + \frac{1}{2} \Psi'_I(x) (nd)^2 + n\frac{1}{2} \Psi'_I(x) d^2 = 0, \quad (25)$$

$$\frac{\partial L}{\partial d} = n \Psi_I(x) nd - n \Psi_V(x) (h - d) = 0. \quad (26)$$

The first equation tells us that one dollar spent in precaution should reduce expected harm and the uncertainty burden of injurer and victims by one dollar. The second equation pertains to the optimal allocation of the loss: this should be shared across the parties according to their disposition towards uncertainty. First-best efficient damages are, therefore, from (26)

$$d^e = \frac{\Psi_V(x^e)}{\Psi_V(x^e) + n\Psi_I(x^e)} h. \quad (27)$$

Given $x^e$, efficient damages increase with the degree of uncertainty aversion of the victims and decrease with the degree of uncertainty aversion of the injurer.\(^{36}\)

\(^{35}\)The same transfer could be used by the policy-maker to address distributional concerns, which are, therefore, out of the picture.

\(^{36}\)This linear sharing rule was first obtained, for the case of simple risk aversion, by Greenwood and Ingene (1980), p. 1061.
If \( n \to \infty \), then \( x^e \) goes to infinity while \( d^e \) goes to nil. If \( x \) is bounded above (\( x \leq x^{\text{max}} \)), then negligence with \( \pi = x^{\text{max}} \) is approximately first-best efficient, as shown by Nell and Richter (2003) for CARA utility functions.

More generally, optimal uncertainty sharing requires that the loss be distributed on all individuals in society (not just injurer and victims), in proportion to their ability to tolerate uncertainty. If the loss is spread on an infinite number of individuals, we turn to the characterization of the first best provided by Shavell (1982), which prescribes \( 1 + p'(x) nh = 0 \) and full insurance for all risk-averse parties. This outcome could be achieved by combining safety regulation and social insurance. Such a policy, however, is outside of the scope of the present paper.

A4. Generality of results. The approximation used in the paper applies if losses are small. The comparison between strict liability and negligence turns on the comparison of the risk premia (Conditions I and V). For non-small losses, these conditions can be generalized as follows.

Strict liability dominates negligence if \( \Psi_I(x^n) \leq \Psi_V(x^n) \) for all income levels of injurer and victim. When injurer and victim formulate the same income-independent beliefs, this is the case if the degrees of Absolute Risk and Ambiguity aversion of the victim (\( \rho_V \) and \( \theta_V \), income dependent) are both greater than those of the injurer for any income level. This condition is also met if \( \rho_I \leq \rho_V \) for any income level and the injurer is not subject to ambiguity.

Negligence dominates strict liability with compensatory damages if \( \Psi_V(x^c) \leq \Psi_I(x^c) \) for all income levels of the parties. Again, when injurer and victim formulate the same beliefs, this is the case if: \( \theta_V \leq \theta_I \) and \( \rho_V \leq \rho_I \) for all income levels of the parties or if \( \rho_V \leq \rho_I \) and the victim is not subject to ambiguity.

Let us consider the case with a very large number of victims. Strict liability imposes a risk with infinite variance on the injurer, while negligence imposes a risk with bounded variance on the victims. So, unless the injurer is risk-neutral, risk spreading (negligence) is the optimal policy. Considering different beliefs and monotonically transforming expected utilities (ambiguity aversion) does not revert this result.
References


