Cooperation on Climate-Change Mitigation†

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Abstract

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Abstract
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1. Introduction

Global environmental problems such as climate change, depletion of the ozone layer and loss of biodiversity have risen to the top of the world’s environmental agenda. For each of these problems there is a large scientific literature warning of the dangers of the failure to address the issue and continuing with business-as-usual (IPCC 2007, MA 2005). For climate change, the Intergovernmental Panel on Climate Change (IPCC) concluded that continued emissions of greenhouse gases (GHG) would likely lead to significant warming over the coming centuries with the potential for large negative consequences (IPCC 2007).

Climate change and other global environmental problems, however, are particularly difficult to address precisely because they are global. Global environmental problems require concerted action by numerous sovereign countries where actions impose private costs while generating a global public benefit. Sovereignty of nations implies that any international environmental agreement must be self-enforcing. But designing self-enforcing agreements is problematic given the nature of global public goods because the self interest of each country is best served by having other countries bear the cost of addressing the problem while they free-ride on these efforts.


Another strand of literature analyzes the incentives of countries to participate in international environmental agreements. As the negotiations at the annual Conference of Parties under the UN Framework Convention on Climate Change both pre- and post-Kyoto illustrate, countries can choose to participate or stay on the sidelines (e.g., the U.S. and China). Even if a country chooses
to participate, there are limited sanctions available to punish countries that do not meet their climate change treaty obligations. It is now clear that many countries that agreed to binding emissions caps under the Kyoto Protocol will exceed their caps for the 2008-2012 period but it is unlikely that such countries will face effective sanctions. Addressing questions of whether a country will choose to participate in a climate change agreement, or if participating will choose to comply with an agreement, requires an analysis of the strategic interests of each country involved. Several studies have applied static or repeated games (Chander and Tulkens 1995, Barrett 2003, Finus 2001) or have included the dynamics of GHG stock in a game-theoretic model (Bosello et al. 2003, de Zeeuw 2008, Eyckmans and Tulkens 2002, Rubio and Ulph 2007) to analyze international agreements on climate change. However, these studies focus solely on the stability of an international environmental agreement, analyzing whether any country can improve its payoff by joining or dropping out of the agreement. Treaty member countries are assumed to cooperate fully with the agreement even though cheating may improve a treaty member's welfare. Nordhaus and Yang (1996) investigate a dynamic game but assume that countries adopt open-loop strategies where countries commit to future emissions at the outset of the game so that equilibria are not necessarily subgame perfect. Martin et al. (2003), Mäler and de Zeeuw (1998), and Yang (2003) study a dynamic game allowing for closed-loop strategies, but without including the role of punishment for potential defectors, which limits the ability to achieve efficient outcomes and overcome free-rider problems.

A desirable model of international environmental agreements as applied to climate change would include stock effects (so that the interaction is properly modeled as a dynamic game rather than a static or repeated game), and would allow countries to credibly punish defector states. Such a model would by necessity focus on closed-loop or feedback strategies. To our knowledge, only a few papers include these ingredients. Dockner et al. (1996) and Dutta and Radner (2004, 2005, 2006, 2009) find conditions under which cooperative equilibrium can be supported as a subgame perfect equilibrium through the use of a trigger strategy. In these models, once some country cheats on the agreement, punishment begins and continues forever. In the climate change application, however, punishment under a trigger-strategy would involve mutually assured over-accumulation of GHGs forever. A legitimate criticism of such strategies is that they are not robust against renegotiation once a country's deviation triggers punishment because the countries
can do far better by restarting an agreement. Most international sanctions are in fact temporary in nature.\footnote{Based on 103 case studies of economic sanctions between World War I and 1984, Hufbauer et al. (1985) find that the average length of successful and unsuccessful sanctions were 2.9 and 6.9 years. Success of a sanction is defined in terms of the extent to which the corresponding foreign policy goal is achieved (p.79).} Dutta (1995b), Polasky et al. (2006), and Tarui et al. (2008) analyze how to support efficient use of a common property renewable resource in a subgame perfect equilibrium using a two-part punishment scheme with severe punishment for a short period followed by reversion to the efficient solution. Though extraction from a common property resource shares many similarities with climate change, an important difference is that damages from climate change are potentially unbounded versus in the worst payoff in a resource model where a player can opt of the commons and get a payoff of zero.

In this paper, we analyze the problem of designing a self-enforcing international environmental agreement for climate change as a dynamic game with two-part punishment strategies. In each period, each country chooses its level of economic activity. Economic activity generates benefits for the country but also generates emissions that increase atmospheric GHG concentrations, which negatively affect the welfare of all countries in the current and future periods. The benefit function depends nonlinearly on emissions to reflect increasing marginal abatement cost while the damage from climate change also depends nonlinearly on GHG concentrations. Atmospheric concentrations evolve over time through an increase of concentrations from emissions of GHG and the slow decay of existing concentrations. We analyze a strategy profile in which each country initially chooses emissions that generate a Pareto optimal outcome (first-best or cooperative strategy) and continues to play cooperatively as long as all other countries do so. If a country deviates from the cooperative strategy, all countries then invoke a two-part punishment strategy. In the first phase, countries inflict harsh punishment on the deviating country by requiring it to curtail emissions. In the second phase, all countries return to playing the cooperative strategy. We design the two-part punishment scheme to ensure that the punishment is sufficiently severe to deter cheating and all countries have an incentive to carry out the punishment if called upon to do so.

Our model with nonlinear benefit and damage functions build on existing dynamic-game models by Dutta and Radner (with nonlinear benefits and linear damages) and Dockner et al. (with linear...
benefits and nonlinear damages). While many researchers argue that marginal abatement costs (negative of the marginal benefits of GHG emissions) are highly nonlinear (e.g. Nordhaus and Boyer 2000), scientists predict that the impacts of climate change will be a nonlinear function of GHG concentrations (IPCC 2007). Economic studies of the effect of climate change on agriculture show non-linearity impacts (Schlenker et al. 2006, Schlenker and Roberts 2006). Nonlinearity may also arise due to catastrophic events such as the collapse of the thermohaline circulation (THC) in the North Atlantic Ocean (Broecker 1997). We find that the degree of nonlinearity in both benefits and damages plays a significant role in the ability to support an efficient self-enforcing international agreement on climate change.

We identify conditions under which a two-part punishment strategy can support the first-best outcome as a subgame perfect equilibrium, i.e., when a self-enforcing international environmental agreement can generate an efficient outcome. Though such conditions are necessarily complicated for dynamic games with nonlinear functions, we exploit a few properties of the game to simplify the analysis. We provide a simulation model to illustrate conditions under which it is possible for a self-enforcing agreement to support an efficient outcome. We also parameterize the simulation model to mimic current conditions to show whether a self-enforcing agreement that achieves optimal climate change policy is likely to be possible. We find that whether the two-part punishment strategy leads to a self-enforcing optimal policy depends non-monotonically on the discount factor. While sufficiently small discount factors do not make cooperation supportable for the standard reason that the threat of future punishment is insufficient to deter the current benefit from cheating, sufficiently large discount factors may also make cooperation too costly to support. Dutta (1995b) found that monotonicity regarding the discount factor—an implication of the folk theorem for repeated games—does not carry over to dynamic games. Our finding is consistent with this general result for dynamic games. We also find non-monotonicity regarding the marginal benefits and costs of GHG emission reduction: the two-phase treaty does not support cooperation when the slope of the marginal benefits of GHG emission is too large or too small relative to the slope of the marginal abatement cost. Our linear-quadratic example illustrates that these non-linear relationships are relevant over the range of values of key parameters—discount factor, elasticity of marginal utility, marginal benefits and costs of GHG emission reduction—that are used in previous numerical studies on climate change. Our dynamic-game analysis also demonstrates that
the treaty may become supportable as the atmospheric concentration of GHG becomes sufficiently large while the treaty is not supportable at lower concentration levels.

In what follows, section 2 describes the assumption of the game, the treaty design, and a few properties of the game that simplifies the conditions under which the treaty is a subgame perfect equilibrium. Using an example with quadratic functions, section 3 discusses the condition under which the two-part strategy profile is a subgame perfect equilibrium. In Section 4, we choose the parameter values of the quadratic functions based on previous climate-change models to illustrate the implication to climate-change mitigation. After summarizing the implication of countries’ heterogeneity in Section 5, we conclude the paper in Section 6.

2. Basic model

2.1 Assumptions

In each period, \( t = 0,1,\ldots \), each country, \( i = 1,\ldots,N \), chooses a GHG emission level, \( x_{it} \), \( 0 \leq x_i \leq \bar{x} < \infty \), where \( \bar{x} \) is the maximum feasible emission level for country \( i \). The transition of the GHG stock in the atmosphere is given by

\[
S_{i,t+1} = g(S_i, X_i) = S + \lambda(S_i - S) + \beta X_i,
\]

where \( X_i = \sum x_{it} \), \( 1 - \lambda \) represents the natural rate of decay of GHG per period \( (0 < \lambda < 1) \), \( S \) is the GHG stock level prior to the industrial revolution, and \( \beta \) is the retention rate of current emissions \( (0 \leq \beta \leq 1) \).\(^2\) Let \( x_{-it} \) be a vector of emissions by all countries other than \( i \) and let \( X_{-it} = \sum_{j \neq i} x_{jt} \), the total emissions by all countries other than \( i \).

We denote the payoff to country \( i \) in period \( t \) with emission \( x_{it} \) when the GHG stock is \( S_i \) by \( \pi_i(x_{it}, S_i) \). We assume that each country \( i \)'s period-wise payoff equals the economic benefit from emissions, \( B_i \), which is a function of its own emissions, minus climate damage, \( D_i \), which is a function of the current GHG stock:

\(^2\) Many studies have used this specification of GHG stock transition (Nordhaus and Yang 1996, Newell and Pizer 2003, Karp and Zhang 2004, Dutta and Radner 2004).
\[ \pi_i(x_i, S_i) = B_i(x_i) - D_i(S_i). \]

We assume that \( B_i \) is a strictly concave function with \( B_i(0) = 0 \), that it has a unique maximum \( x_i^b > 0 \), and that \( B_i'(x) > 0 \) for all \( x \in (0, x_i^b) \). We call \( x_i^b \) the “myopic business-as-usual” (myopic BAU) emission level of country \( i \). This level of emissions maximizes current benefits from emissions without taking into account the damages associated with contributions to the stock of GHGs. We assume the damage function is increasing and convex in the GHG stock, \( D_i' > 0, D_i'' > 0 \).

Countries have the same one-period discount factor \( \delta \in (0,1) \). The discount factor \( \delta \) incorporates the growth rate of benefits and damages (see section 4.1 for more discussions about the discount factor). We assume that countries have complete information (i.e., there is no uncertainty) and that each country observes the history of GHG stock evolution and all countries' previous emissions.

### 2.2 Efficient solution

The efficient emissions path solves the problem.

\[
\max \sum_{t=0}^{\infty} \delta^t \sum_i \pi_i(x_i, S_i)
\]

\( s.t. \quad S_{t+1} = g(S_t, x_t) \quad \text{for } t = 0, 1, \ldots \text{ given } S_0. \)

The solution to this problem generates a sequence of emissions \( \{x_i^*\}_{i=0}^{\infty} \) where \( x_i^* = \{x_i^*\}_{i=1}^N \). The corresponding value function solves the functional equation. In what follows, we suppress time subscripts to simplify notation. The dynamic programming solution for this problem solves:

\[
V(S) = \max_x \sum_i \pi_i(x_i, S) + \delta V(S')
\]

\( s.t. \quad S' = g(S, x). \)

We assume the solution is interior. The optimal emission profile given \( S \),
\[
x^*(S) = \{x_1^*(S), x_2^*(S), \ldots, x_N^*(S)\}, \]

satisfies

\[
\frac{\partial \pi_i(x_i^*(S), S)}{\partial x_i} + \delta V'(g(S, X^*(S)))\beta = 0
\]

for all \( i \). The first term represents the marginal benefit of emissions in country \( i \) while the second
term is the discounted present value of the future stream of marginal damages in all countries from the next period. Thus, under the efficient allocation, the marginal abatement costs of all countries in the same period are equal, and equal to the shadow value of the stock.

The unique steady state $S^*$ satisfies:

$$\frac{\partial \pi_i(x_i^*(S^*), S^*)}{\partial x_i} + \frac{\delta \beta}{1 - \delta \lambda} \frac{\partial \sum_j \pi_j(x_j^*(S^*), S^*)}{\partial S} = 0.$$  

Given $S_0 < S^*$, the stock increases monotonically to the steady state $S^*$. For the rest of the paper we assume $S_0 < S^*$. In what follows, we describe a strategy profile that supports $x^*$ as a subgame perfect equilibrium.

### 2.3 A strategy profile to support cooperation

Consider the following strategy profile $\phi^*$, which may support efficient emissions reduction with a threat of punishment against cheating.\(^3\)

**Strategy profile $\phi^*$**

- **Phase I**: Countries choose $\{x_i^*\}$. If a single country $j$ chooses $x_j \neq x_j^*$, with resulting stock $S'$, go to Phase II $(S')$. Otherwise repeat Phase I in the next period.
- **Phase II $(S')$**: Countries play $x^j = (x_1^j, \ldots, x_j^j, \ldots, x_N^j)$ for $T$ periods. If a country $k$ deviates with resulting stock $S''$, go to Phase II $(S'')$. Otherwise go back to Phase I.

The idea of the punishment strategy $x^j$ is to have country $j$ (that cheated in the previous period) choose low emissions for $T$ periods while the other countries choose high emissions. The punishment for country $j$ in Phase II works in two ways, one through its own low emissions (and hence low benefits during Phase II) and the other through increases in its future stream of damages due to an increase in the other countries' emissions during Phase II. The sanction is temporary, and countries resume cooperation once the sanction is complete. Under some

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\(^3\) The design of the two-part punishment scheme to support cooperation is similar to those discussed in Abreu (1988).
parameter values and with appropriately specified penal codes \( \{x^j\} \), no country has an incentive to cheat against the cooperative agreement, and if some country does cheat, all countries have an incentive to carry out the sanction. We now turn to a discussion of the condition under which \( \phi^* \) is a subgame perfect equilibrium.

**Sufficient conditions for efficient sustainability**

Let \( V^C_j(S, I) \) be country \( j \)'s present value of payoffs given \( \phi^* \) with cooperation in the current period, and \( V^D_j(S, I) \) be country \( j \)'s present value of payoffs given \( \phi^* \) with the maximum payoff upon deviation in Phase I given current stock \( S \). Similarly, let \( V^C_j(S, II_i), V^D_j(S, II_i) \) be \( j \)'s present value of payoffs upon cooperation and an optimal deviation starting in Phase II given stock \( S \). The strategy profile involving cooperation and a two-part punishment scheme is a subgame perfect equilibrium if the following conditions are satisfied for all \( j, j = 1, \ldots, N \):

**Condition (1)** Country \( j \) has no incentive to deviate in phase I: \( V^C_j(S, I) \geq V^D_j(S, I) \);

**Condition (2)** Country \( j \) has no incentive to deviate in phase II \( j \): \( V^C_j(S, II_j) \geq V^D_j(S, II_j) \); and

**Condition (3)** Country \( j \) has no incentive to deviate in phase II \( k \) for all \( k \neq j \):

\[
V^C_j(S, II_k) \geq V^D_j(S, II_k);
\]

for all possible stock levels given initial stock \( S_0 \). Because each player's periodwise return is bounded from above and the discount rate is positive, the principle of optimality for discounted dynamic programming applies to this game. Hence, in order to prove that \( \phi^* \) is subgame perfect, it is sufficient to show that any one-shot deviation cannot be payoff-improving for any player (Fudenberg and Tirole 1991). Because this is a dynamic game, we need to verify that no player has an incentive to deviate from the prescribed strategy in any phase and under any possible stock level.\(^4\)

Because combined emissions are nonnegative and bounded by the maximum feasible level \( \bar{X} \equiv \sum_i \bar{x}_i \), the feasible stock levels lie between 0 and \( \bar{S} > 0 \) where \( \bar{S} \) satisfies

\[^4\] See Dutta (1995a) for a similar analysis in a dynamic game context.
\[ \bar{S} = S + \lambda (\bar{S} - S) + \beta \bar{X} \]
\[ \bar{S} = S + \frac{\beta \bar{X}}{1 - \lambda} \]

where \( \bar{X} = \sum_{i=1}^{N} x_i \). We can exploit a few properties to simplify the above three conditions for an efficient outcome. Let \( x_i^D(S) \) be the optimal deviation that maximizes country i’s payoff upon deviation in either phase I or II. Under a reasonable assumption that \( x_i^D \) exceeds the efficient emission level, it turns out that the three sufficient conditions can be reduced to one, namely condition (2). (Proofs of all propositions are presented in the appendix.)

**Proposition 1** Suppose the punishment phase lasts one period, \( T=1 \), \( x_j^f \equiv z > 0 \) is independent of the post-deviation stock level, \( x_i^D(S) > x_j^x(S) \), and \( \sum x_j^i(S) = X^*(S) \) given any S. Then condition (2) implies conditions (1) and (3).

Proposition 1 shows that it is sufficient to evaluate condition (2) for a particular form of the two-part punishment scheme where the country that has deviated emits a fixed level \( z \) and where other countries’ emissions are designed so that combined emissions equal the efficient level given the post-deviation stock. While this strategy need not be optimal, focusing our discussion on this strategy greatly simplifies the task of verifying the existence of a strategy profile that yields a subgame perfect equilibrium. The next result shows that strategy profiles with multiple period punishments can be evaluated in a similar fashion.

**Proposition 2** Country \( j \) has no incentive to deviate in any period during phase II \( j \) if it has no incentive to deviate in the first period in phase II \( j \).

Proposition 2 extends Proposition 1 to penal strategies with multiple periods of punishment. Since country \( j \) may cheat in any period during Phase II\( j \) in this context, the concern is that there are now many conditions to check. The proposition shows that it is sufficient to check condition (2) for the first period of Phase II\( j \).
Because there is an upper bound on stock, there are at most three candidate values for stock that could yield maximal deviation gains during phase II. If gains are monotonic in stock, the argmax is either the lowest possible value\(^5\) or the largest possible value. If gains are non-monotonic, then either there is an interior maximum (in which case the relevant stock value is that value which delivers the interior maximum) or there is an interior minimum, in which case the argmax is one of the two corners. In any event, it becomes a simple matter to check that deviation gains in phase II\(_j\) are never positive; this insight leads to the next result.

**Proposition 3** Let \(S^m\) be the stock at which gains from deviation in phase II\(_j\) are maximized. If these gains are non-negative, the two-part punishment strategy \(\phi^*\) yields a subgame perfect equilibrium.

When \(\phi^*\) is not a subgame perfect equilibrium, there may be another strategy profile that supports the efficient as a subgame perfect equilibrium outcome. A punishment is most effective as deterrence against over-emitting if it induces the over-emitter’s minmax (i.e. the worst perfect equilibrium) payoff. Though such punishment supports cooperation under the widest range of parameter values, a two-part punishment scheme inducing the worst perfect equilibrium may be too complicated to generate useful insights about self-enforcing treaties. Previous dynamic game studies have analyzed cooperation with worst perfect equilibria in the context of local common-property resource use (Dutta 1995b, Polasky et al. 2006). With local common-property resource use, the minmax level is defined by outside options for resource users—the payoffs that they would receive if they quit resource use. With a global commons problem such as climate change, there are no outside options: a country cannot escape from changed climate. With linear damage functions, Dutta and Radner (2004) find that the worst perfect equilibria take a simple form (constant emissions by all countries). With nonlinear damage functions, the worst perfect equilibria will be more complicated because they may depend nonlinearly on the state variable. In

\(^5\) Certainly pollution stocks cannot be negative. Plausibly, there is a subsistence level of economic activity (that induces an associated subsistence level of emissions). Any country emitting less than that amount for an indefinite period of time would preclude its survival (though it might be able to survive for short periods, as during a punishment phase). If so, the lower bound on stock is strictly positive, and conceivably equal to the initial value.
this study, we restrict our attention to $\phi^*$, a strategic profile with a simple two-part punishment scheme, in order to gain insights about countries’ incentives to cooperate in a treaty with simply designed sanctions.

3. An example with homogeneous countries
To illustrate our proposed strategy we analyze a simple concrete example. In this example we assume that each of the $N$ countries' period-wise return functions are linear-quadratic:

$$\pi_i(x_i, S) = ax_i - bx_i^2 - cS - dS^2,$$

where $a, b, c, d > 0$. The negative of the derivative with respect to emissions, $-a - 2bx_i$, represents the marginal abatement cost associated with emissions $x_i$. Country $i$’s myopic BAU emission that maximizes the period-wise return is $x_i^b = x^b \equiv a/(2b)$. As shown in the appendix, the value function is quadratic and a unique linear policy function exists for the efficient solution. The values $2b$ and $2d$ represent the slopes of the marginal costs of emission reduction and the marginal damages from pollution stock.

Next, we assume that the maximum feasible emission is given by $\bar{x}_i > 0$ defined by $B(\bar{x}_i) = 0$, i.e. $\bar{x}_i = a/b$ for all $i$. Thus, $\bar{S} = \frac{Na}{b(1-\lambda)}$. Finally, we assume that the initial stock is smaller than the potential steady state stock: $S_0 \leq S^*$. We simplify the stock transition by setting $S = 0$ and $\beta = 1$.

The strategy profile we investigate takes a particularly simple form: $x_j^i(S) \equiv z > 0$, a constant smaller than $x^*(S^*)$, for all $S \geq 0$ and all $j$; $x_j^i(S) \equiv y(S) \equiv \frac{X^*(S) - z}{N-1}$ for all $i \neq j$; and $T=1$.

With this two-part punishment scheme, all countries $i \neq j$ choose the optimal aggregate emissions $X^*(S)$ collectively for all $S$.

[Figure 1]
Figure 1 illustrates a case where $\phi^*$ is a subgame perfect equilibrium. In each panel, the solid curve represents the payoff upon cooperation while the dotted curve represents the payoff upon optimal deviation. This figure is based on the parameterization $a = 10, b = 500, c = 0 = z, d = 0.001, N = 4$ and $\delta = .99 = \lambda$. The optimal steady state GHG stock is around 3.45 while the maximum feasible stock $\bar{S}$ equals 8. Under the assumed combination of parameter values, the payoffs upon cooperation exceed the payoffs upon optimal deviation under all relevant stock levels in each phase.

We now turn our attention to an analysis of the role played by the various parameters.

3.1 The influence of exogenous parameters on supportability of cooperation

We start with a discussion of the influence of the discounting factor upon the supportability of cooperation. Figure 2 illustrates the main points. The key parameters in this figure are the same as in Figure 1, though here we vary $d$ between 0.001 and 0.01 and $\delta$ between 0.95 and 0.99. In this figure, the shaded area represents the combinations of the discount factor and the slope of marginal damages where treaty supportability holds (i.e. $\phi^*$ is a subgame perfect equilibrium). While in repeated games cooperative outcomes are only supportable if players are sufficiently patient (an implication of the folk theorem), this need not be true in dynamic games (Dutta, 1995b). For our dynamic game, supportability of $\phi^*$ as a subgame perfect equilibrium need not be monotonic in the discount factor: Indeed; for values of $d$ (the slope of marginal damage resulting from the GHG stock) between .003 and .010 the profile $\phi^*$ is not supportable when $\delta$ is too close to 1 or less than .96. A key factor leading to this non-monotonicity result is that the efficient, optimal emissions and the associated optimal stock transition change as the discount factor changes while the optimal actions would stay constant in repeated games. When $\delta$ is too small, cooperation is not supportable because the future payoff associated with cooperation is discounted too heavily. As $\delta$ increases, the payoff associated with cooperation increases while the first-best emission level decreases. Therefore, both the future payoff associated with cooperation and the payoff associated with optimal deviations increase. Movement along the arrow in Figure 2 indicates that the latter may increase by a larger amount than the former when the discount factor is sufficiently large. We note also that larger values of $d$ allow cooperation to be supported at smaller discount
factors. We now delve deeper into the role played by marginal damages, in particular as they relate to marginal abatement costs.

[Figure 2]

The key parameters in Figure 3 are the same as in the preceding figures, though here we vary \( b \) between 1 and 1,000 and \( d \) between 0.00001 and 0.0004. The crucial value here is \( b/d \), the ratio of the slope of marginal abatement costs, \( b \), to the slope of marginal damages, \( d \). As Figure 3 illustrates, \( \phi^* \) is a subgame perfect equilibrium when this ratio is neither too large nor too small. At points like H, where \( b/d \) is large, the magnitude of marginal damages from the GHG stock is small relative to marginal abatement costs. Here, the difference between the optimal emissions and noncooperative emission levels are small, which makes the potential gains from cooperation relatively small. For smaller values of \( b/d \), marginal damages increase faster than the marginal abatement costs as pollution stock increases. Accordingly, the difference between the optimal emissions and noncooperative emission levels becomes larger. Because the optimal emission control calls for larger emission reduction to each country, both the gains from cooperation and temptations to deviate increase. When \( b/d \) is not too small, the former exceeds the latter and \( \phi^* \) supports cooperation. However, at points like L, the temptation to deviate exceeds the gains from cooperation and hence \( \phi^* \) is not a subgame perfect equilibrium.

[Figure 3]

4. Illustration with parameter values for climate change

The preceding discussion highlights two key parameters that play a crucial role in determining whether a two-part penal code can generate a subgame perfect equilibrium: the discount factor, \( \delta \), and the ratio of the slope of the marginal abatement costs to the slope of marginal damages, \( b/d \).

What does our model predict regarding the supportability of cooperation based on parameter values employed in existing economic studies on climate change? We apply our game theoretic model with linear-quadratic specification in order to address these questions. Several previous studies on climate change apply the linear-quadratic specification (e.g. Newell and Pizer 2003
Karp and Zhang 2006, 2009). Though simplistic, linear-quadratic models have several advantages. They ease the burden of computing equilibria of dynamic games, and they are “simple to calibrate and easy to interpret, making it possible to understand the effect of assumptions about parameters” (Karp and Zhang 2006). Given that there is a wide range of uncertainty about the parameters, we consider a range of key parameter values used in the existing literature.

4.1 Parameter assumptions

To obtain value of \( b/d \), we start with the ratio of slopes at the global level, which we denote by \( \gamma \). With \( N \) identical countries, this parameter depends on \( N, b \) and \( d \) via \( \gamma = b/(dN^2) \). We consider a range of values for \( \gamma \) including those used in the previous studies (e.g. Nordhaus and Boyer 2000, Newell and Pizer 2003, Karp and Zhang 2009).

Following Nordhaus and Yang (1996) and Newell and Pizer (2003), we specify decay rate of CO\(_2\) to be .83\% and the retention rate of current emissions to be 64\% (per year). The pre-industrial stock level is \( S = 613 \) GtC (gigatons of carbon equivalent), while the initial stock level \( S_0 = 787 \) GtC (the level in 1995). Thus the carbon stock transition is given by

\[
S_{t+1} = 0.9917(S_t - 613) + 613 + 0.64X_t,
\]

We assume that country \( i \)'s net benefit from emissions and the damage from stock are proportional to country \( i \)'s national income:

\[
B(x_{it}) = y_i \bar{B}_i(x_{it}), \quad D(S_t) = y_i \bar{D}_i(S_t),
\]

where \( \bar{B} \) and \( \bar{D} \) represent the net benefit and damage as fractions of income, respectively.

Assuming income grows at the rate \( 1 + g_y + n \), where \( g_y \) is the per capita income growth rate and \( n \) the population growth rate, and denoting \( y_0 \) denote initial income, the net benefits for country \( i \) in year \( t \) are

\[
\Pi_i(x_{it}, S_t) = y_0 (1 + g_y + n)^t \bar{B}_i(x_{it}) - y_0 (1 + g_y + n)^t \bar{D}_i(S_t),
\]

With \( N \) identical countries, the ratio of slopes of global marginal abatement cost to global marginal damages is \( \gamma = (b/N)/(Nd) \). According to United Nations (2007), the world population in 2000 is 6,124.213 million while the median forecast of the population in 2050 is 9,191.287 million. Hence, the average annual population growth rate between 2000 and 2050 is about 8.2\%.
With a discount factor $\bar{\delta}$, the present value of country $i$'s payoff stream is

$$\sum_{t=0}^{\infty} \bar{\delta}^t \Pi(x_t, S_t) = \sum_{t=0}^{\infty} \bar{\delta}^t \left[ y_0(1+g_y+n)B(x_t) - y_0(1+g_y+n)\overline{D}(S_t) \right]$$

Then letting $r$ represent the consumption discount rate, the effective discount is given by

$$\delta = \bar{\delta}(1+g_y+n)/(1+r).$$

In the Ramsey formula under optimal growth, $r = \rho + \eta g_y$, where $\rho$ is the rate of time preference, $g_y$ is the growth rate of consumption, and $\eta$ is the elasticity of marginal utility. The effective discount factor in our model is therefore

$$\delta = (1+g_y+n)/(1+\rho + \eta g_y).$$

A number of recent studies discuss what values should be used for these parameters when analyzing climate change (Stern et al. 2006, Nordhaus 2007, Dasgupta 2007). The values adopted in these studies imply a range of values for the discount rate.

While there are over 190 countries in the world, the lion’s share of emissions can be attributed to a small number of countries. Indeed; China, the European Union, India, Japan, Russia and the United States are collectively responsible for roughly three-quarters of global CO2 emissions (Netherlands Environmental Assessment Agency, 2008). Except as noted, the simulations we report below are based on a model with six countries, and so can be interpreted as reflecting the large-scale GHG emitters.

We extend the earlier discussion by allowing $z$ to vary with $S$: $z(S) = \alpha x^*(S)$, where $0 \leq \alpha < 1$ and $x^*(S)$ is the efficient emission per country given stock $S$. The parameter $\alpha$ can be interpreted as a measure of the severity of punishment: if country $j$ cooperates in phase II, it would reduce the emissions by $100(1-\alpha)$ percent relative to the efficient level. As before, we assume that each other country chooses $[X^*(S) - z(S)]/(N-1)$ in phase II. In phase II, they collectively choose $X^*(S) - z(S)$, so that collectively these countries choose $[X^*(S) - z(S)]/(N-1)$. As in much of the above discussion, we assume $T=1$.

### 4.2 Findings

Figure 4(a) illustrates supportability of the penal code for a range of the values of $\gamma$ adopted from the literature. The figures assumes $\lambda = 0.9917$, $N = 6$, $S = 613$GtC, $N = 6$, $a = 0.0074*(B/D)$, $T=1$, $D=$
6.0195*10-4, C= -0.8526 and $\gamma \equiv B/D$ between 5,000 and 200,000. The parameter $\delta$ varies between 0.97 and 0.9999 in panel (a) and is fixed at $\delta=1/1.01$ in panel (b). The shaded area indicates the set of parameter value combinations where the treaty is a subgame perfect equilibrium for some value of $\alpha$. The red lines indicate three values of $\gamma$ used in the literature: $\gamma=12,269$ (Karp and Zhang 2009), 53,630 (Nordhaus and Boyer 2000), and 183,908 (Newell and Pizer 2003). Other studies have used estimates of $\gamma$ of the same order of magnitude (e.g. Hoel and Karp 2002, Falk and Mendelsohn 1993, Reilly 1992). See the appendix for a brief survey of the estimates of gamma used in the literature. The figure has several implications regarding the economics of climate change. First, non-monotonicity regarding the discount factor (recall Figure 2) is not only a theoretical possibility but may be relevant in the context of climate change. Non-monotonicity might occur (i.e. an increase in discount factor makes the penal code non-supportable) for some values of $B/D$ between the two used in Newell and Pizer (2003) and Nordhaus and Boyer (2000). Secondly, the penal code is not supportable for low values of $B/D$ even though a lower value of $B/D$ would certainly justify an aggressive climate-change mitigation as an optimal policy. Therefore, a decrease in $B/D$—due to more optimistic estimates of the slope of the marginal abatement costs or more pessimistic estimates about the slope of the marginal damages—does not necessarily imply that cooperation becomes more supportable. (Though not described in the figure, the treaty is not supportable when $B/D$ is very large.)

[Figure 4]

Panel (b) of Figure 4 describes the range of $\alpha$ where the treaty is supportable for a fixed discount factor $1/1.01$ and for the same range of values of $B/D$ as in panel (a). The shared area represents the values of $\alpha$ where the treaty is supportable for each level of $B/D$. The figure shows that the self-enforcing severity of punishment greatly varies over the range of $B/D$.

**Discount rate, elasticity of marginal utility, and income growth rate**

Since the Stern Review of the Economics of Climate Change was published, economists have been debating over the choice of discount rates and the other parameter values in the Ramsey formula (e.g. Nordhaus 2007a, b, Weitzman 2007, Dasgupta 2007, Heal 2009). The following exercise illustrates the implication of the choice of these parameter values on supportability of the proposed
treaty.

Recall the formula for the effective discount factor in this model:

\[ \delta = \frac{(1 + g_y + n)}{(1 + \rho + \eta g)}. \]

Figure 5 describes treaty supportability when all parameter values are fixed except for \( \rho \), the rate of time preferences, and \( \eta \), the elasticity of marginal utility. The figures assume \( \lambda = 0.9917 \), \( N = 6 \), \( \xi = 613 \text{GtC} \), \( a = 0.0074 \times (B/D) \), \( T = 1 \), \( D = 6.0195 \times 10^{-4} \), \( C = -0.8526 \).

[Figure 5]

This figure also assumes \( g_y = 2\% \) and \( n = 0.82\% \) (the annual population growth rate between 2000 and 2050 based on the United Nations median forecast\(^7\)). Each panel assumes a value of \( \gamma \) identified in Figure 3. The dotted areas represent the combinations of \( \rho \) and \( \eta \) where the effective discount factor \( \delta \) is larger than one, and hence our model is not well defined. For example, \((\rho, \eta) = (0.001, 1)\) —the assumption used in Stern Review—falls in this area. Panel (a) represents a case where the treaty is not supportable when \( \rho \) and \( \eta \) are both too small or too large.

Figure 6 illustrates the supportability of treaty when all parameter values are fixed except for \( \rho \) and \( g_y \). The two panels correspond to the two values of \( B/D \) considered in Figure 5. Each panel describes treaty supportability for three values of the elasticity of marginal utility. In all panels, the treaty-supporting set of \( \rho \) and \( g_y \) appears to shrink as \( \eta \) increases. All panels indicate a trade-off between \( \rho \) and \( g \) when \( \eta \) is large (2 or 3).

[Figure 6]

How does the number of players influence the first-best sustainability? Some dynamic games with sanctions predict monotonicity regarding the number of players (i.e. cooperation is supportable as a subgame perfect equilibrium outcome with \( N \) players if it is with the number of players smaller

\(^7\) According to United Nations (2007), the world population in 2000 is 6,124.213 million while the median forecast of the population in 2050 is 9,191.287 million. Hence, the average annual population growth rate between 2000 and 2050 is about 8.2%.
A common-property resource use game in Polasky et al. (2006) is an example where this monotonicity holds. On the other hand, monotonicity does not necessarily hold for other dynamic games with sanctions (e.g. Tarui et al. 2008). The intuition is that the cost-benefit ratio of sanctions may decrease as the number of players increases because the cost burden of sanctions for each sanctioning player may decrease faster than the rate at which the benefit of sanctions for each player decreases.

Figure 7 describes the relation between supportability of the penal code and the number of countries. The figures assumes $\lambda = 0.9917$, $S = 613 \text{GtC}$, $a = 0.0074(B/D)$, $T = 1$, $D = 6.0195 \times 10^{-4}$, $G = -0.8526$, $\delta = .995$. For a given value of the global ratio $\gamma = B/D$, we set $b = B/D$ and $d = 1/N^2$ (see footnote 8). In this case and other cases considered, monotonicity regarding the number of countries always held.

5. Extension to heterogeneous countries

How does heterogeneity across countries influence supportability of $\phi^*$? In this section we sketch out a variation of our model to investigate this question. To this end, we assume the punishment phase is characterized by $x^i_j(S) \equiv 0$ for all $S \geq 0$ and all $j$ and $x^i_j(S) \equiv x^h_i$ (the myopic BAU emission) for all $i \neq j$. We retain the earlier assumption that $S_0 \leq S^*$. We allow transfers among countries. Let $\tau_{it}$ be the net transfer to country $i$ in period $t$ where $\sum_t \tau_{it} = 0$ for all $t$. Country $i$’s net one-period return in period $t$ is given by $\pi_i(x_{it}, S_t) + \tau_{it}$. In the context of climate-change mitigation, the transfers would be determined based on cost burden sharing agreed on by the countries.

Suppose the value to country $i$ is given by $\sigma_i V_i$ where $\sigma_i \geq 0$ and $\sum_i \sigma_i = 1$. Define a transfer $\tau^*$ where

$$
\tau^*_i(S) \equiv \sigma_i \sum_j \pi_j(x^*_j(S), S) - \pi_i(x^*_i(S), S)
$$

for all $S$ and all $i$. By choosing $x^*$ and $\tau^*$, the countries realize the efficient outcome with the
shares induced by $\sigma$.

[Figure 8]

Figure 10 describes the gains from cooperation under the treaty for a game with 4 heterogeneous countries. The figure assumes $\lambda = .99$ and $v_i = V/N$ for all $i$, $a_i = 10.2$ ($10.0$) for countries with high (low) BAU emissions, $d_i = .0001$ (.00008) for countries with a high (low) $b/d$ ratio, $b_i = 5$ for all $i$ and $c=0$. The figure also assumes $v_i = V/N$ for all $i$, i.e. the payoffs upon cooperation are divided equally across countries. With this example, conditions (1) and (3) under which $\phi^*$ is a subgame perfect equilibrium holds given the discount factor values considered (2.5%, 10%). The steady state is about 320 when $\delta=1/1.1$ and 222 when $\delta=1.1025$. The treaty is supportable when the discount rate is 2.5%. With a 10% discount rate, the gains from cooperation becomes negative—first for Country 4 with large BAU emissions (i.e. larger value of $a_i/2b_i$) and high b/d ratio. then for Country 3 with low BAU but high b/d. Given equal sharing of $V$, a country with a higher BAU emission and lower marginal damages has less to lose by deviation than countries with lower BAU emissions and higher marginal damages. This example illustrates different incentives for controlling emissions by countries with different benefits and damages.

6. Discussion

Climate change mitigation is a global public good where reducing GHG reduction is costly for each country while GHG stock accumulation in the atmosphere is likely to cause damages to many countries. In order for sovereign countries to cooperate through an international agreement to control GHGs, the agreement must be self-enforcing for each country. We applied a dynamic game to illustrate an international agreement with a simple rule of sanctions in order to support the efficient, cooperative climate-change mitigation. Instead of a trigger strategy where all countries choose over-emissions forever upon some country’s cheating, we considered a two-part penal code where countries resume cooperation after completing a temporary sanction against an over-emitter. With numerical examples and illustrations using a simple climate-change model, we examined the conditions under which such a simple two-part sanction scheme—where the country being sanctioned chooses a low emission and the others choose over-emissions for a finite
period—is a subgame perfect equilibrium.

Our numerical example confirmed that each country's incentive to cooperate may change as the stock level changes. We might expect that it may become easier for countries to avoid free riding and cooperate as GHG stock increases; however, we found that a larger stock level does not necessarily imply that the sanction scheme is more likely to be a subgame perfect equilibrium.

We also find that whether the two-phase treaty is self-enforcing depends nonlinearly on the countries’ discount factor: while sufficiently small discount factors do not make cooperation supportable, sufficiently large discount factors may also make cooperation too costly to support. We also find non-monotonicity regarding the marginal benefits and costs of GHG emission reduction: the two-phase treaty does not support cooperation when the slope of the marginal benefits of GHG emission is too large or too small relative to the slope of the marginal abatement cost. Our linear-quadratic example illustrates that these non-linear relationships are relevant over the range of values of key parameters—discount factor, elasticity of marginal utility, marginal benefits and costs of GHG emission reduction—that are used in previous numerical studies on climate change.

Our findings imply that dynamic-game formulation provides a useful framework for analyzing a self-enforcing treaty for climate-change mitigation and a useful insight that may not be available from static-game or repeated-game analysis. The game-theoretic approach taken in this paper—with key assumptions including nonlinear damages and emission abatement costs, limited-term sanctions, and feedback strategies used by countries—will be useful for further analysis of self-enforcing treaties. Heterogeneity among countries and its implications toward treaty design should be further explored. Countries differ in costs and benefits regarding climate-change mitigation as well as the greenhouse gas emission levels in the past and in the future. How such countries should share the cost of GHG emission reduction is a crucial issue (Germain et al. 2003, Chakravarty et al. 2009). Future research will investigate the role of transfers and the self-enforcing transfer mechanisms.  

8 Yang and Nordhaus (2006) address the issue of technology transfers in the context of climate change mitigation.
Future research should also address a number of assumptions that we made to keep our analysis simple. We assumed no technological progress and the same income and population growth rates for all countries in the analytical model. Other natural extensions of our model would be to incorporate uncertainty regarding climate change, and to consider sanctions through means other than increased emissions such as trade (Barret 2003, Lockwood and Whalley 2008). A temporary trade sanctions may be less costly for each country than sanctions with increased emissions, the effect of which will last for a long time because of the nature of GHG as a stock pollutant. Future research may study the extent to which the availability of trade sanctions increases the likelihood of a self-enforcing treaty.

Appendix 1

Conditions for the efficient outcome

Suppose $T = 1$. Consider countries' incentive to deviate in Phase I. In period $t$, given current stock $S$, country $j$'s payoff upon cooperation is $V_j(S_j)$. Country $j$'s payoff upon deviation, with over-emission $x_j^D$ in period $t$, is given by

$$
\pi_j(x_j^D, S) + \delta \pi_j(x_j^D(S_{r+1}), S_{r+1}) + \delta^2 V_j(S_{r+2})
$$

where $S_{r+1} = g(S, x_j^D, x_j^D) + S_{r+1}' = g(S_{r+1}, x_j^*(S_{r+1}))$. This is a discounted sum of a current gain by over-emitting in period $t$, a low return in period $t+1$ due to punishment, and continuation payoffs with a larger GHG stock due to its own over-emission. Therefore, no country deviates from Phase I if

$$
V_j(S) \geq \pi_j(x_j^D, S) + \delta \pi_j(x_j^D(g(S, x_j^D, x_j^D(S))), g(S, x_j^D, x_j^D(S))) + \delta^2 V_j(g(g(S, x_j^D, x_j^D(S)), x_j^D(g(S, x_j^D, x_j^D(S))))
$$

for all $x_j^D \geq 0$, $S \geq S_0$ and all $j$.

In Phase II, country $j$ does not deviate from cooperation if

$$
\pi_j(x_j^D(S), S) + \delta V_j(g(S, x_j^D(S))) \geq \pi_j(x_j^D(S), S) + \delta \pi_j(x_j^D(g(S, x_j^D, x_j^D(S))), g(S, x_j^D, x_j^D(S))) + \delta^2 V_j(g(S, x_j^D, x_j^D(S)), x_j^D(g(S, x_j^D, x_j^D(S))))
$$

for all $x_j^D \geq 0$ and all possible stock levels given $S_0$ (i.e. for all $S \geq g(S_0, x_j^D(S_0), x_j^D(S_0)))$.

Similarly, in Phase II, country $j$ has no incentive to deviate if
\[
\pi_j(x_j^*(S), S + \delta V_j(g(S, x^*(S)))) \geq \pi_j(x_j^0(S), S) + \delta \pi_j(x_j^0(g(S, x^0_j(S)), g(S, x^0_j, x^0_j))) + \delta^2 V_j(g(S, x^0_j, x^0_j))
\]
for all \( x^0_j \geq 0 \) and all \( S \geq g(S_0, (x^0_j, x^0_j)) \).

To summarize, the strategy profile is a subgame perfect equilibrium if conditions (1), (2), and (3) (for all \( k \neq j \)) hold for all possible deviations, all \( S \geq S_0 \) and all \( j \).

**Proof of Proposition 1**

Let \( \hat{x} \) be the optimal deviation by country \( i \) in phase II, \( \nu \) the optimal value function of country \( i \), and \( X^* \) the optimal total emission of \( N \) countries (all of which are functions of stock \( S_i \).) Let \( G(z, S_i) \) be the gain from deviation for country \( i \) in Phase II, \( (S_i) \):

\[
G(z, S_i) = B(\hat{x}) - D(S_i) + \delta \left[ B(z) - D(S_{i+1}) \right] + \delta^2 \nu(S_{i+1}) - \left[ B(z) - D(S_i) + \delta \nu(S_{i+1}) \right]
\]

where \( S_{i+1} = \lambda S_i + X^*(S_i) \), \( z + \hat{x}, S_{i+1} \equiv \lambda S_i + X^*(S_i) \), and \( S_{i+1} \equiv \lambda S_i + X^*(S_i) \).

**Step 1.** To show \( \frac{\partial G}{\partial z} = B'(\hat{x}) - (1 - \delta)B'(z) \).

The partial derivative of \( G \) with respect to \( z \) is given by

\[
\frac{\partial G}{\partial z} = B'(\hat{x}) \frac{\partial \hat{x}}{\partial z} - B'(z) + \delta B'(z) - \delta D'(S_{i+1}) \left[ -1 + \frac{\partial \hat{x}}{\partial z} \right] + \delta^2 \nu'(S_{i+1}) \left[ \lambda + X^*(S_{i+1}) \right] - \left[ B(z) - D(S_i) + \delta \nu(S_{i+1}) \right]
\]

where the first term equals zero due to the envelope theorem. Therefore,

\[
\frac{\partial G}{\partial z} = - \left[ B'(\hat{x}) - \delta D'(S_{i+1}) + \delta^2 \nu'(S_{i+1}) \left[ \lambda + X^*(S_{i+1}) \right] \right] + B'(\hat{x}) - (1 - \delta)B'(z).
\]

Applying the envelope theorem once again, we have

\[
\frac{\partial G}{\partial z} = B'(\hat{x}) - (1 - \delta)B'(z).
\]

Let \( \hat{x}_j(S_i) \) be the optimal deviation by country \( i \) in phase I, \( S_i \) and \( G_j(z, S_i) \) the gain from deviation for country \( i \) in Phase I, \( (S_i) \):

\[
G_j(z, S_i) = B(\hat{x}_j) - D(S_i) + \delta \left[ B(z) - D(S_{i+1}) \right] + \delta^2 \nu(S_{i+1}) - \left[ B(z) - D(S_i) + \delta \nu(S_{i+1}) \right]
\]

where \( S_{i+1} \equiv \lambda S_i + X^*(S_i) \), \( \lambda S_i + X^*(S_i) + \hat{x}_j, S_{i+1} \equiv \lambda S_i + X^*(S_i) \), and \( S_{i+1} \equiv \lambda S_i + X^*(S_i) \).

**Step 2.** To show \( \frac{\partial G_j}{\partial z} = \delta B'(z) \).
The partial derivative of \( G \) with respect to \( z \) is given by

\[
\frac{\partial G}{\partial z} = B'(\hat{x}) \frac{\partial \hat{x}}{\partial z} + \partial B'(z) - \partial D'(S^{d}_{i+1}) \frac{\partial \hat{x}}{\partial z} + \delta^2 v'(S^{d}_{i+2}) \left[ \lambda + X''(S^{d}_{i+1}) \right] \frac{\partial \hat{x}}{\partial z}
\]

\[
= \frac{\partial \hat{x}}{\partial z} \left[ B'(\hat{x}) - \partial D'(S^{d}_{i+1}) + \delta^2 v'(S^{d}_{i+2}) \right] + \partial B'(z) = \partial B'(z),
\]

where the last equality follows from the envelope theorem.

Step 3. To show \( G(z, S) \geq G_{f}(z, S) \) for all \( S \).

Note that \( G(x^*(S), S) - G_{f}(x^*(S), S) = 0 \), i.e. the gains from deviation are the same in Phases I and II when \( z \) equals \( x^*(S) \). Steps 1 and 2 imply

\[
\frac{\partial}{\partial z} [G(z, S) - G_{f}(z, S)] = B'(\hat{x}) < 0
\]

for all \( z \in [0, x^*(S)] \) given any stock level as long as \( \hat{x} > z \). It follows from the last two equations that \( G \geq G_{f} \) for all \( z \). Hence, condition (2) implies condition (1). A similar argument proves that condition (2) implies condition (3).

**Proof of Proposition 2**

Let \( T \geq 2 \). Suppose that, for any \( S \in [S_0, S^*] \), cooperating in Phase II \( j(S) \) is preferred to optimal deviation in the first period in the phase:

\[
B(z) - D(S) + \delta [B(z) - D(S_1)] + \delta^2 [B(z) - D(S_2)] + \cdots + \delta^{T-1} [B(z) - D(S_{T-1})] + \delta^T v(S_T) \geq B(\hat{x}) - D(S) + \delta [B(\hat{x}) - D(S_1)] + \delta^2 [B(\hat{x}) - D(S_2)] + \cdots + \delta^{T-1} [B(\hat{x}) - D(S_{T-1})] + \delta^T v(S_{T-1})
\]

where \( S_1 = g(S, X^*(S)) \), \( S_2 = g(S_1, X^*(S_1)) \), \( S_1^* = g(S, X^*_j(S) + \hat{x}) \), \( S_2^* = g(S_1^*, X^*(S_1^*)) \), and so on. The payoff upon deviating in the second period in the phase is given by

\[
B(z) - D(S) + \delta [B(\hat{x}_2) - D(S_1)] + \delta^2 [B(z) - D(S_2)] + \cdots + \delta^{T-1} [B(z) - D(S_{T-1})] + \delta^T v(S_{T-1}^2)
\]

where \( S_1^2 = g(S, X^*_j(S) + \hat{x}_2) \), \( S_2^2 = g(S_1^2, X^*(S_2^2)) \), and so on. The inequality follows because the first inequality above holds for \( S_i \in [S_0, S^*] \). Now the last expression would correspond to the payoff upon cooperating in phase II \( j \) if phase II lasted for \( T + 1 \) periods. Because the continuation payoff for country \( j \) in phase II \( j \) is decreasing in the number of periods it lasts (\( T \)), it follows that the last expression does not exceed the payoff upon cooperation in phase II \( j \) when it lasts for \( T \) periods:

\[
B(z) - D(S) + \delta [B(z) - D(S_1)] + \delta^2 [B(z) - D(S_2)] + \cdots + \delta^{T-1} [B(z) - D(S_{T-1})] + \delta^T v(S_T).
\]

That is, if the first-period deviation does not improve country \( j \)'s payoff in phase II \( j \), then the second-period deviation does not improve it either. The same logic applies as we compare the payoffs upon optimal deviation in the first period with those in the third or later periods. Therefore, for phase II \( j \) to be self-enforcing, it is sufficient to verify that the payoff upon cooperation in phase II \( j \) is greater than or equal to the payoff upon optimal deviation when deviation occurs in the first period of phase II \( j \).
Appendix 2 Assumptions of the climate change exercise

Several studies have used quadratic models to approximate the global benefit and damage functions (Falk and Mendelsohn 1993, List and Mason 2001, Newell and Pizer 2003, Karp and Zhang 2005, 2006). We combine these models with an estimate of regional benefits and damages by Nordhaus and Yang (1996) and Nordhaus and Boyer (2000).

Assume that country $i$’s emission benefit and the climate damage in period $t$ are both functions of $y_{it}$, the country's GDP in period $t$:

$$\{\hat{a}_i, \hat{b}_i, \hat{c}_i, \hat{d}_i, \hat{f}_i\} y_{it} - \{\hat{c}_i S_t + \hat{d}_i S^2 + \hat{f}_i\} y_{it},$$

Assume that country $i$’s population growth rate is $n_i > 0$ and the growth rate of GDP per capita is $g_i > 0$. These rates are both constant over time. If the country’s consumption discount rate is $r_i > 0$, then country $i$’s present-value return is given by

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r_i}\right)^t (1+n_i)^t (1+g_i)^t y_{io} \{\hat{a}_i, \hat{b}_i, \hat{c}_i, \hat{d}_i, \hat{f}_i\} y_{it}$$

$$= \sum_{t=0}^{\infty} \delta_i^t y_{io} \{\hat{a}_i, \hat{b}_i, \hat{c}_i, \hat{d}_i, \hat{f}_i\} y_{it},$$

where $\delta_i = \frac{(1+n_i)(1+g_i)}{1+r_i}$, $a_i = y_{io} \hat{a}_i$, $b_i = y_{io} \hat{b}_i$, $d_i = y_{io} \hat{d}_i$, and so on. In this simulation, we assume that the benefit and damage functions are time-invariant except for changes due to GDP and population growth. We also assume that all countries' composite discount factors are identical: $\delta_i = \delta$ for all $i$. Future research will consider the effects of change in these functions due technological changes on countries' incentive for cooperation.

The base year is 1995. A time period corresponds to one year. We model the emissions and accumulation of CO$_2$ and do not consider other GHGs. What follows is a list of assumptions about the parameter values.

**Benefit functions**

We specify the benefit functions using the following functional form:

$$B_i(x_i) = \frac{(\bar{e}_i - x_i)^2}{2\theta_i} y_{it} = \left\{\frac{1}{2\theta_i} x_i - \frac{\bar{e}_i}{\theta_i} x_i^2 - \frac{\bar{e}_i^2}{2\theta_i}\right\} y_{it},$$

where $\bar{e}_i > 0$ represents country $i$’s myopic BAU emission and $y_{it}$ the regional GDP. The value $1/\theta_i$ represents the slope of the marginal cost of emission reduction (in terms of % of GDP) for country $i$. This specification allows us to aggregate the benefit functions to derive the global...
benefit in a simple way. A function \( B \) where \( B(X) \equiv -\frac{(E - X)^2}{2\Theta} \) with \( E = \sum_i \bar{e}_i \), and \( \Theta \equiv \sum_i (\theta_i/y_{i0}) \) satisfies

\[
B(X) = \max_{\{x_i\}} \left\{ \sum_i -\frac{(\bar{e}_i - x_i)^2}{2\theta_i} \left| \sum x_i = X \right. \right\}.
\]

This function \( B \) represents the global net benefit function. We chose the values of \( \{\bar{e}_i\} \) to be equal to the CO\(_2\) emissions from the corresponding regions in the year 1995. Nordhaus and Yang (1996) use an estimate of \( \{B_i(0)/y_{i0}\} \), the fraction of annual GDP required to reduce net CO2 emissions to 0 in each region:

\[
B_i(0)/y_{i0} = \frac{\bar{e}_i^2}{2\theta_i}.
\]

We can pin down the value of the parameters \( \theta_i \), \( a_i \), \( b_i \), and a constant term \( f_{li} \) using the estimates of the right-hand side and \( \bar{e}_i \):

\[
a_i = y_{i0} \frac{1}{2\theta_i}, \quad b_i = y_{i0} \frac{\bar{e}_i}{\theta_i}, \quad f_{li} = y_{i0} \frac{\bar{e}_i^2}{2\theta_i}.
\]

Table A-1 summarizes the regional aggregates assumed for our simulation. Figure 4-1 draws the regional marginal abatement costs of CO2 emissions. While the marginal abatement costs are relatively high in Japan and Europe, they are lower in US, Russia, and China. The relative ranking of MAC across regions agrees with the estimates in existing studies. Figure 4-2, which lists the marginal abatement cost estimate by Ellerman and Decaux (1998), shows a very similar ranking of MACs.

**Damage functions**

We specify the following functional form for the damage functions.

\[
D_i(S_i) = y_{i0} \{d_{li}(S_i - S)^2 + c_i(S_i - S) + f_{2i}\}.
\]

Nordhaus (1998) estimates the regional damages, in terms of a fraction of regional GDP, as a function of changes in the average atmospheric temperature relative to the preindustrial level:

\[
D_i^T(T) = d_{li}T + d_{2i}T^2.
\]

He derives the values of the parameters \( \{d_{li}, d_{2i}\} \) by calibrating the above function to the point estimates of damages at 2.5 and 6 mean temperature increases (see Table 4-1). Following Kattenberg et al (1996) and Newell and Pizer (2003), assume that temperature change is proportional to the change in the log of the carbon stock. We pin down the CO2 stock level corresponding to a 2.5 increase by using this log relationship: This implies that the carbon stock level associated with 2.5 warming is

\[
T(S) \approx 2.885 \ln \left( \frac{\bar{S}}{S} \right).
\]

---

9Specifically, let \( T(S) \) be the increase in temperature, relative to the preindustrial level, associated with the CO\(_2\) concentration \( S \). Then

\[
T(S) \approx 2.885 \ln \left( \frac{\bar{S}}{S} \right).
\]
Because we assume a linear-quadratic damage as a function of CO2 stock, we apply linear interpolation to identify the damage function parameters:

\[
D_i(S) = d_{i1}(T(S)) + d_{i2}(T(S))^2 = d_{i1}(t(S - S)) + d_{i2}(t(S - S))^2
\]

\[
= d_{i1}tS - d_{i1}tS + d_{i2}t^2(S^2 - 2SS + S^2) = S^2(d_{i2}t^2) + S(d_{i1}t - 2Sd_{i2}t^2) - d_{i1}tS + d_{i2}t^2 S^2.
\]

We pin down the value of \( t \) by plugging in \( D_i(S_{2.5}) = D^f_i(2.5) \) and \( S = S_{2.5} \). Once \( t \) is identified, we can find the values of the damage function parameters:

\[
d_i = d_{i2}t^2, \quad c_i = d_{i1}t - 2Sd_{i2}t^2, \quad f_i = -d_{i1}tS + d_{i2}t^2 S^2.
\]

The constant term in \( i \) is identified by \( f_i = f_{ii} + f_{2i} \).

Figure A-3 represents the damage as % of regional GDP for each region. According to Nordhaus’s estimates, India, Europe, and Africa are more vulnerable to climate change than US, China, and Russia. Some countries including Russia is expected to experience positive climate benefits for a modest increase in the CO2 stock.

With the above assumptions, the ratio \( B/D \) based on Nordhaus and Boyer’s model (2000) is

\[
\frac{B}{D} \approx \frac{32.3827}{6.0195 \times 10^{-4}} \approx 53,630.
\]

Table A-1 Data used for simulation.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td>6,892</td>
<td>0.851</td>
<td>-0.00095</td>
<td>0.00491</td>
<td>0.05</td>
</tr>
<tr>
<td>USA</td>
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<td>0.07</td>
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<td>MI</td>
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<td>0.00390</td>
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<td>OHI</td>
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<td>-0.01079</td>
<td>0.00369</td>
<td>0.05</td>
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<td>China</td>
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<td>0.00201</td>
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<tr>
<td>LI</td>
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<td>0.327</td>
<td>0.00628</td>
<td>0.00249</td>
<td>0.10</td>
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<tr>
<td>India</td>
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<td>0.00492</td>
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<td>Russia</td>
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<td>-0.01078</td>
<td>0.00327</td>
<td>0.15</td>
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<td>HIO</td>
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<td>Africa</td>
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<td>0.01560</td>
<td>0.00097</td>
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<td>World</td>
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<td>6.159</td>
<td>0.00071</td>
<td>0.00270</td>
<td>0.07</td>
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</tbody>
</table>

Note: (A), (B): Nordhaus and Boyer (2000), Table 3.2, p. 39. (C) (D): Nordhaus 1998 Table 12, (E): Nordhaus and Yang (1996), Table 2, p. 746. (A) is measured in billion 1990 US GDP with market exchange rates and (B) in billion metric tons of carbon equivalent. Parameters d1, d2 are damage function parameters such that D1T+D2T^2 represents regional damage (percentage of regional GDP) when temperature increase is given by T. C1 is an abatement cost function parameter and represents the fraction of annual output required to reduce net CO2 emissions to 0.

Table A-2 summarizes the estimates of \( \gamma \) used in the literature.

Table A-2 Estimates of $\gamma$ in previous studies.

<table>
<thead>
<tr>
<th>Study</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karp and Zhang (2009)</td>
<td>12,269</td>
</tr>
<tr>
<td>Newell and Pizer (2003)</td>
<td>183,908</td>
</tr>
<tr>
<td>Nordhaud and Boyer (2000)</td>
<td>53,630</td>
</tr>
<tr>
<td>Hoel and Karp (2002) point estimate</td>
<td>71,114</td>
</tr>
<tr>
<td>FM as discussed in HK(2002)</td>
<td>72,993</td>
</tr>
<tr>
<td>Reilly as discussed in HK (2002)</td>
<td>50,000</td>
</tr>
</tbody>
</table>

References


Individual payoffs in phase I

Payoffs in phase IIj for player j

Steady state (3.45)

Figure 1: An example where $\phi^*$ is a subgame perfect equilibrium.
Figure 2: Discount factor and treaty supportability.
Supportability of the penal code

Figure 3: Marginal abatement costs, marginal damages, and treaty supportability.
(a) Supportability of Treaty

γ (ratio of the slope of marginal abatement costs to the slope of marginal damage) x 10^5

δ (discount factor)

γ = 12,269
γ = 53,630
γ = 183,908

Shaded area: penal code is supportable

(b) Sanction parameter and treaty supportability

α (sanction parameter)

Range of supporting α

γ = 12,269
γ = 53,630
γ = 183,908

Figure 4: Supportability of cooperation.
The discount factor is greater than or equal to 1.

Figure 5: Time preference, elasticity of marginal utility, and treaty supportability.
Figure 6: Time preference, income growth rate, and treaty supportability.
Figure 7: Supportability of cooperation and the number of countries.
Figure 8: Gains from cooperation for heterogeneous countries.
Notes on the efficient solution of a quadratic example (with heterogeneity allowed)

Suppose country $i$'s periodwise return in period $t$ is given by

$$a_i x_i - b_i x_i^2 - (c_i S_t + d_i S_t^2 + f_i),$$

given emissions $x_i$ and stock $S_t$ where $a_i, b_i, d_i > 0$ for all $i$. (The parameters $\{c_i, f_i\}$ may take positive or negative values.) Let $a = (a_1, a_2, \ldots, a_n)$, $b = (b_1, b_2, \ldots, b_n)$, and $d = (d_1, d_2, \ldots, d_n)$ be column vectors. Let $C = \sum c_i$ and $F = \sum f_i$. Let $B$ be an $N$ by $N$ diagonal matrix whose diagonal entries are $b$. Given stock $S$ and an emissions profile $x$, the total periodwise return of $N$ countries is

$$a'x - x'Bx - (CS + DS^2 + F),$$

where $D = \sum d_i$. The state transition is given by

$$S_{t+1} = \lambda S + \beta'x + h$$

where $\lambda \in (0,1)$, $\beta$ a column vector where $1 - \beta_i$ represents the short-run decay rate, and $h \equiv (1 - \lambda)S$ is a constant. Let $V$ be the total value function of all $N$ countries:

$$V(S) = \max_x a'x - x'Bx - (CS + DS^2 + F) + \delta V(S')$$

s.t. $S' = \lambda S + \beta'x + h$. Because $V$ is quadratic, let $V(S) = PS^2 + QS + R$ where $P, Q, R$ are scalars. We have

$$V(S') = (\lambda S + \beta'x + h)'P(\lambda S + \beta'x + h) + Q(\lambda S + \beta'x + h) + R$$

$$= (\lambda S + h)'P(\lambda S + h) + (\lambda S + h)'P\beta'x + x'\beta P(\lambda S + h) + x'\beta P\beta'x + Q\lambda S + Q\beta'x + Qh + R.$$

So

$$\frac{\partial}{\partial x} V(S') = \beta P(\lambda S + h) + \beta P(\lambda S + h) + 2\beta P\beta'x + Q\beta$$

$$= 2\beta P(\lambda S + h) + 2\beta P\beta'x + Q\beta.$$

The first order condition is

$$a - 2Bx + \delta[2\beta P(\lambda S + h) + 2\beta P\beta'x + Q\beta] = 0.$$

Hence $[2B - 2\delta P \beta']x = a + 2\delta \beta P(\lambda S + h) + \delta Q\beta$, i.e.

$$x = (1/2)[B - \delta P \beta']^{-1}[a + 2\delta \beta P(\lambda S + h) + \delta Q\beta].$$

Substitute this expression into the functional equation and we obtain the following expression:

$$P = -D + \delta Q^2 P - \delta^2 \lambda S + \beta^2 \Phi(P) \beta,$$

where $\Phi(P) \equiv -[B + \delta P \beta']^{-1}$. Solve this function for $P$, and we can solve for the remaining unknown $Q$ and $R$.

$$Q = \frac{-\delta P \lambda a' \Phi(P) \beta - C - 2\delta^2 P^2 \lambda h \beta' \Phi(P) \beta + 2\delta \lambda h P}{1 + \delta^2 P \lambda \beta' \Phi(P) \beta - \delta \lambda},$$

$$R = \frac{1}{1 - \delta} \left\{ -\frac{1}{4} (a' + \delta Q \beta') \Phi(P)(a + \delta Q \beta) - (a' + \delta Q \beta') \Phi(P) \beta - \Phi(P) \delta \beta P h 
- \delta^2 P^2 \beta' \Phi(P) \beta h^2 + \delta Q h - F + \delta h^2 P \right\}. $$

NOTE (Not for publication) 1
**Optimal deviation**

Assume \( T = 1 \), \( x_i'(S) = \alpha x_i^*(S) \) where \( 0 \leq \alpha < 1 \), and

\[
x_i'(S) = x_i^*(S) + \sum_{k \neq i} x_i^*(S) (1 - \alpha) x_j'(S).
\]

Note that

\[
x_i'(S) + \sum_{k \neq i} x_i^*(S) = \alpha x_i^*(S) + \sum_{k \neq i} x_i^*(S) + (1 - \alpha) x_j'(S) = X^*(S),
\]

for all \( S \). Hence the total emission in Phase II \( j(S) \) equals the socially optimal aggregate emission given stock \( S \).

The optimal deviation given \( S \) solves the following problem.

\[
\max_{x \geq 0} a_i x - b_i x^2 - (c_i S + d_i S^2 + f) + \delta \left[ a_i \alpha x_i^*(S_i) - b_i \{ \alpha x_i^*(S_i) \}^2 - (c_i S_i + d_i S_i^2 + f) \right] + \delta^2 v_i(S_2),
\]

where \( S_i = \lambda S + \beta X_i + h = \lambda S_i + \beta (\Theta S_i + \Gamma) + h = (\lambda + \beta \Theta)(\lambda S + \beta X_i + h) + (\lambda + \beta \Theta) \beta x + \beta \Gamma + h \).

The first-order condition is

\[
0 = a_i - 2b_i x + \delta \left[ a_i \alpha \frac{\partial x_i^*(S_i)}{\partial S_i} \frac{\partial S_i}{\partial x} - 2b_i \alpha^2 x_i^*(S_i) \frac{\partial S_i}{\partial x} - \left( 2 \lambda S_i + \frac{\partial S_i}{\partial x} + c_i \frac{\partial S_i}{\partial x} \right) \right] + \delta^2 \left[ 2 p_i \{ (\lambda + \beta \Theta) \lambda S + \beta (\lambda + \beta \Theta) \beta x + \beta \Gamma + h \} + q_i \frac{\partial S_i}{\partial x} \right]
\]

\[
= a_i - 2b_i x + \delta \left[ a_i \alpha \theta_i - 2b_i \alpha^2 (\theta_i (C_i + \beta x) + \gamma_i) \theta_i \beta - (2d_i (C_i + \beta x) + c_i \beta) \right] + \delta^2 \left[ 2 p_i \{ (\lambda + \beta \Theta) C_i + (\lambda + \beta \Theta) \beta x + \beta \Gamma + h \} + q_i \{ \lambda + \beta \Theta \} \beta \right],
\]

where \( C_i = \lambda S + \beta X_i + h \).

Arrange the terms and we have

\[
0 = a_i + \delta \left[ a_i \alpha \theta_i \beta - 2b_i \alpha^2 (\theta_i (C_i + \gamma_i) + \gamma_i) \theta_i \beta - (2d_i (C_i + \gamma_i) + g_i \beta) \right] + \delta^2 \left[ 2 p_i \{ (\lambda + \beta \Theta) C_i + \beta \Gamma + h \} + q_i \{ \lambda + \beta \Theta \} \beta - 2b_i x + \delta \left[ 2 \lambda S_i + \frac{\partial S_i}{\partial x} + c_i \frac{\partial S_i}{\partial x} \right] \right] + \delta^2 \left[ 2 p_i \{ (\lambda + \beta \Theta) C_i + (\lambda + \beta \Theta) \beta x + \beta \Gamma + h \} + q_i \{ \lambda + \beta \Theta \} \beta \right].
\]

Solving the condition for \( x \), we obtain country \( i \)'s optimal deviation \( x_i^d \):

\[
x_i^d(S) = a_i + \delta \left[ a_i \alpha \theta_i \beta - \lambda S_i + \gamma_i \theta_i \beta - (2d_i (C_i + \beta x) + g_i \beta) \right] + \delta^2 \left[ 2 p_i \{ (\lambda + \beta \Theta) C_i + \beta \Gamma + h \} + q_i \{ \lambda + \beta \Theta \} \beta \right] = \frac{a_i + \delta \left[ a_i \alpha \theta_i \beta - \lambda S_i + \gamma_i \theta_i \beta - (2d_i (C_i + \beta x) + g_i \beta) \right] + \delta^2 \left[ 2 p_i \{ (\lambda + \beta \Theta) C_i + \beta \Gamma + h \} + q_i \{ \lambda + \beta \Theta \} \beta \right]}{2b_i + \delta \left[ 2b_i \alpha^2 (\theta_i \beta - (2d_i \beta \beta) \right] - \delta^2 \left[ 2 p_i \{ (\lambda + \beta \Theta) \} \beta \right]}.\]

Note that \( C_i \) depends on the stock level as well as the phase in which country \( i \) deviates:
\[ C_i = \begin{cases} 
\lambda S + \beta \left\{ \sum_{j \neq i} x_j^*(S) \right\} + h, & \text{in Phase I}(S); \\
\lambda S + \beta \left\{ (1 - \alpha) x_i^*(S) + \sum_{j \neq i} x_j^*(S) \right\} + h, & \text{in Phase II}_1(S); \\
\lambda S + \beta \left\{ X^*(S) - x_i^*(S) - \frac{x_i^*(S)}{\sum_{j \neq k} x_j^*(S)} (1 - \alpha) x_k^*(S) \right\} + h, & \text{in Phase II}_k(S). 
\end{cases} \]