Advantageous Selection in Insurance Markets*

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Abstract

This paper reverses the standard conclusion that asymmetric information plus competition results in insufficient insurance provision. Risk-tolerant individuals take few precautions and are disinclined to insure, but are drawn into a pooling equilibrium by the low premiums created by the presence of safer, more risk-averse types. Taxing insurance drives out the reckless clients, allowing a strict Pareto gain. This result depends on administrative costs in processing claims and issuing policies, as does the novel finding of a pure-strategy, partial-pooling, sub-game-perfect, Nash equilibrium in the insurance market.

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1. Introduction

Such empirical evidence as we have appears to conflict with the major implications of the standard economic model of insurance. For example, 4.8% of U.K. credit cards are reported lost or stolen each year whereas for insured cards the corresponding figure is only 2.7%. In similar vein, Cawley and Philipson (1999) find that the mortality rate of US males purchasing life insurance is below that of the uninsured, even when controlling for many factors, such as income, which are correlated with life expectancy. Chiappori and Salanie (2000) establish a methodology for such studies and report that controlling for observable characteristics known to insurers, the accident rate of young French drivers choosing comprehensive insurance is lower than for those opting for the legal minimum coverage, although the difference is not statistically significant.

These findings contradict the predictions of models of insurance markets under asymmetric information, as initiated and exemplified by Rothschild and Stiglitz (1976). The basic idea of models in this tradition is that the incentive to purchase insurance is greatest for those having private information that they are relatively likely to suffer a loss. Augmenting this adverse-selection effect is moral hazard, the tendency of insurance to dull the incentive to take precautions, thereby intensifying the loss propensity of the insured relative to that of the uninsured.

This paper adopts a different perspective. It drops the assumption that people have identical risk preferences but differ in the level of exogenously determined risk they are exposed to. Instead, our starting point is that cautious people are not only more inclined to buy insurance, but also put more effort into limiting risk exposure than those of a more reckless disposition. This formulation poten-

\[^1\]The first figure is supplied by APACS, the credit-card issuers representative body, and the second from an insurance company which does not wish to disclose its identity.
tially explains the evidence that the insured are less accident prone. In addition, the standard welfare conclusion that there is insufficient insurance provision is reversed. The conventional under-insurance result arises because companies anticipate a self-selection bias and so set high premiums, making it unattractive for good risks to take out policies, even though they would be more than willing to pay the actuarially fair price for their characteristics. In our model, the presence of cautious types lowers premiums and thus draws into the market relatively risk-tolerant, reckless types. Taxing insurance purchase drives out the bold types, eliminating a negative externality and permitting a strict Pareto gain. This result, and indeed the existence of equilibrium itself, depends on the presence of administrative costs in processing claims. These are not only realistic, but help provide a way round the celebrated result of Rothschild and Stiglitz (1976) concerning the non-existence of pooling equilibria.

Hidden heterogeneity in risk preferences has been looked at in an insurance market with a monopoly provider by Landsberger and Meilisjon (1994), but this set up does not yield major changes to the conclusions of the standard model. The combination of hidden preventative activity and hidden types is the potent combination and has been discussed to some extent by Pauly (1974). More recent analyses are Stewart (1994) and Chassagnon and Chiappori (1997), both of which explicitly examine equilibria in which agents differ with regard to the cost and effectiveness of preventative effort. Using a version of the Rothschild and Stiglitz model with this feature, Chassagnon and Chiappori show the existence of a positive-profit separating equilibrium, but in their set up, sub-game perfect Nash pooling equilibria do not exist.

Wambach (1997) is rather closer to our model. Exogenous, unobservable wealth differences coexist with unobservable exogenous loss probabilities. Partial-pooling equilibria are shown to arise, possibly involving positive profits. This is
related to our existence result, though we endogenise the correlation between insurance purchase and precautionary behaviour and introduce administrative costs. Jullien, Salanie and Salanie (2000) is similar to our work in that heterogeneous risk preferences drive both precautionary action and insurance choices, although the particular preference formulation is distinct. The major distinction though is that in their model there is a single principal, in effect monopoly provision, whereas we adopt a competitive setting. The positive results are in the same spirit as ours, but, in common with Wambach there is no welfare analysis. Monopoly involves a quite different set of externalities to competition and the conventional result of underprovision is to be expected.

Both our existence and our policy results depend on positive administrative costs, which are indeed significant in practice. Between 1985 and 1995 for UK insurers, expenses as a percentage of premium income averaged 25 per cent for motor insurance and 37 per cent for property damage insurance.\footnote{Data from Association of British Insurers.}

The other key ingredient of our analysis is that precautionary effort is positively correlated with insurance purchase. For example, controlling for observable characteristics, our approach implies that buyers of accidental death insurance are cautious types who will experience lower than average accident rates. Evidence along these lines has already been reported.

To see more explicitly how these features fit together, suppose there are equal numbers of two types of potential client, the timid or risk averse, $T$, and the bold or reckless, $B$. The value each puts on a particular insurance policy and the cost of providing it are as shown in Table 1. This example has the property that the

\footnote{Cawley and Philipson (1999) report that the premium per dollar of coverage falls with the level of coverage. They attribute this bulk discounting to fixed costs of underwriting but our model predicts that it is also an implication of the highly insured being the good risks.}
bold value insurance less despite having higher expected claims.

(Table 1. here)

Assume that the insurance industry is competitive and that an individual’s type is private information. If the contract in question is the only one offered, there is evidently a pooling equilibrium in which both types are insured and pay a premium of $75. This is despite the fact that type Bs value the policy less than the cost of providing it to these so that it would be socially efficient not to supply them. Indeed, suppose that every policy carried a tax of $22, with the proceeds distributed as a lump-sum subsidy to the whole population. There is then a separating equilibrium, with premium $82, in which only Ts are insured, but both groups are strictly better off. The Bs lose their expected surplus of $5 from the policy but gain $11 from the poll subsidy, whereas the Ts tax inclusive premium rises by $7 but they gain $11 from the poll subsidy. Everyone gains from intervention.4

In this example there is, by assumption, only one policy and its payout is taken as given so only the premium is to be determined. Whether equilibrium policies really exist and what form they take is a notoriously delicate matter. The remainder of the paper formally demonstrates the existence of pooling, partial pooling and separating equilibria exhibiting over-insurance even when contractual form is endogenous. To make the case as clearly as possible, we adopt the simplest assumptions capable of yielding the novel results, but it should be clear that this stripped down specification is not necessary to obtain our conclusions.

4A monopolist would charge $85, which in this example, maximises aggregate surplus.
2. The Model

Two justifications are offered for the positive correlation between insurance purchase and precautionary activity. The first follows from heterogeneous wealth and lays the foundation for the particular form of heterogeneous tastes that constitutes our second justification.

Suppose, first, that everyone has the same opportunity to lower the probability of a given financial loss through undertaking preventative effort. In the two-state case, the expected utility of an insured individual $i$ is:

$$EU_i(F_i, y, \lambda, W_i) = p(F_i) U(W_i - y) + (1 - p(F_i)) U(W_i - D + \lambda y) - F_i.$$ (1)

where $W_i$ is the person’s wealth, $D$ is the gross loss, $y$ is the insurance premium and $\lambda y$, $\lambda > 0$, the net of premium payout in the event of loss. $F_i$ is a binary choice variable which affects the probabilities of loss in the same way for all individuals. If $F_i = 0$, the probability of avoiding the loss $p(F_i)$ is $p_0$, but if $F_i = F$ the probability rises to $p_F$. The wealth dependent part of the utility function exhibits decreasing absolute risk aversion. This standard assumption implies that the marginal rate of substitution between $y$ and $\lambda y$ falls with wealth. Given the magnitude and probability of loss, lower insurance coverage is therefore chosen by wealthier individuals.

The increase in expected utility from taking precautions is

$$\Delta_i = (p_F - p_0) (U(W_i - y) - U(W_i - D + \lambda y)) - F.$$ (2)

It follows from decreasing absolute risk aversion that if insurance cover is partial, $(D - \lambda y > y)$, then $\partial \Delta_i/\partial W_i < 0$. According to this formulation, there may be a wealth threshold above which precautions are not taken. Moreover, if admin-
istractive costs or other reasons lead to high loading factors, wealthy individuals may prefer to be uninsured.

Now consider a reinterpretation involving differences in preferences. Intuitively, more timid types may lower their risk exposure through increased insurance purchase and greater precautionary effort. However, the concept of a pure change in risk aversion is ambiguous; changing the curvature of the utility function alters its height almost everywhere and the issue is, where should the pivot occur? In general results are ambiguous, but suppose that the utility function of individual $i$ is $U_i = U_i(\alpha_i + W) - F_i$, where $\alpha_i$ is an individual-specific parameter making taste differences formally equivalent to wealth differences. Just as a rich person is less likely than a poor person to take unfair insurance against the loss of $100 and to expend effort to reduce the chance of its loss, this formulation embodies the view that ‘bold’ people behave as if they were much wealthier than they really are. They buy less insurance and take fewer precautions than those with a more ‘timid’ disposition.\(^5\) In what follows we analyse market equilibrium in the heterogeneous taste formulation. Similar results apply for the heterogeneous wealth case.

Assume two types of individual, $T$ and $B$, both equally wealthy. $Bs$ have a high $\alpha$ and so exhibit “bold” behaviour whilst $Ts$ are more “timid”, reflecting a low $\alpha$. For simplicity, but without affecting the qualitative results, we now suppose the special case that at sufficiently high $\alpha + W$, the utility function becomes linear and $Bs$ are in this zone of risk-neutrality with respect to income. In the relevant

\(^5\)Julien, Salanie and Salanie (1999, 2000) also examine whether more risk-averse agents take more precautions. They analyse the purchase of safety enhancements whereas we consider individuals’ choice of precautionary effort. In either case, general results are ambiguous, so the question must ultimately be resolved empirically. The formulation that a poor but risk tolerant individual behaves in the same way as a wealthier but more risk averse person, does yield the intuitive result.
range, the utility functions are:

\[ EU_i (F_i, y_i, \lambda_i, W) = p (F_i) U_i (W - y) + (1 - p (F_i)) U_i (W - D + \lambda y) - F_i, \quad i = T, B; \]

(3)

where \( U_B \) is linear and \( U_T \) is strictly concave and \( W - y \geq W - D + \lambda y \) are the wealth levels in the good and bad states.\(^6\) Given the formulation in (3), the expected utility from taking precautions is:

\[ \Delta_i = (p_F - p_0) [U_i (W - y) - U_i (W - D + \lambda y)] - F, \quad \text{with } \Delta_T > \Delta_B. \]

(4)

There are at least two insurance companies and they incur a strictly positive processing cost, \( C \), per claim handled.\(^7\) Insurers cannot observe the characteristics or verify the actions of individual applicants, but do know the distribution of characteristics among the population of applicants. The expected profit, \( \pi \), of an insurance company selling a contract on which the probability of a claim is \((1 - p (F_i))\) is given by

\[ \pi = p (F_i) \lambda y - (1 - p (F_i)) (\lambda y + C) \]

(5)

2.1. Equilibrium

The structure of the game is as follows:

\(^6\)In principle, the magnitude of loss is endogenous. For example, the cost of accidental damage depends on the type of car owned. This complicates the modelling but does not invalidate our insights.

\(^7\)An important component of \( C \) is monitoring expenditure, so it is in principle endogenous and may vary with the level of coverage. For present purposes this is an inessential complication. In addition, there may be fixed costs of issuing policies. This will introduces an element to \( C \) which is independent of the loss probability but makes negligible difference to the modelling.
Stage 1 Insurance companies make irrevocable offers of contracts that specify premium $y$, and payout $\lambda y$ in the event of loss.

Stage 2 Clients apply for at most one contract from one insurance company. If two insurance companies offer the same contract, clients toss a fair coin to decide between them. In the light of the contract chosen, the client decides whether to take unobservable precautions.

In what follows we only consider pure-strategy, sub-game-perfect, Nash equilibria. Depending on parameter values, separating, full-pooling and partial-pooling equilibria are possible. We begin by outlining the requirements for the various kinds of equilibria.

2.1.1. Separating Equilibrium

A separating equilibrium involves the $T$s and $B$s ending up with distinct allocations, $z_T$ and $z_B$ respectively. Such an equilibrium must satisfy the following:

(a) Incentive compatibility:

\begin{align}
EU_T (z_T) &\geq EU_T (z_B), \\
EU_B (z_B) &\geq EU_B (z_T). \tag{6a}
\end{align}

(b) Effort incentives:

\begin{align}
F_i &= \overline{F} \text{ if } \Delta_i \geq 0, \\
F_i &= 0 \text{ if } \Delta_i < 0. \tag{6b}
\end{align}

with $\Delta_i$ defined in (4).

(c) Participation: For each type $i = B, T$, if they buy insurance, the contract
they subscribe to is at least as good as the null contract, $z_0$.

$$EU_i(z_i) \geq EU_i(z_0),$$  \hspace{1cm} (6c)

(d) Profit Maximisation: Given the contracts offered by the other companies, no company can increase its expected profit by varying the terms of the contracts it offers, or by not offering a contract at all.

2.1.2. Pooling Equilibrium

In a full-pooling equilibrium only the contract $z_p$ offered and everybody buys it. In a partial-pooling equilibrium, $z_p$ attracts at least some of each type of agent but not everyone in the population buys it. Of those applying for $z_p$, a proportion $\rho$ are of type $T$. In both cases, the equilibrium satisfies the following:

(a) Effort incentives:

$$F_i = \overline{F} \text{ if } \Delta_i \geq 0,$$

$$F_i = 0 \text{ if } \Delta_i < 0.$$  \hspace{1cm} (7a)

(b) Participation: For both types, $z_p$ is at least as good as the null contract, $z_0$,

$$U_i(z_p) \geq U_i(z_0), i = B, T;$$  \hspace{1cm} (7b)

In the partial pooling case 7(b) must hold with equality for at least one group.

(c) Profit maximisation: Given that $z_p$ is offered by other companies, no company can increase its expected profit by introducing a different contract or by not offering a contract at all.
2.2. Anatomy of Equilibria

It is easiest to proceed diagrammatically. In all the figures, the individuals' wealth-endowment, \((W, W - D)\), denoted by \((\overline{H}, \overline{L})\), is labelled \(E\). The upward sloping locus \(PP'\) shows the values of \((H, L)\) yielding \(\Delta_T = 0\) and so partitions the space into the lower region where \(Ts\) take precautions, and the upper region where they do not. More explicitly, the slope and curvature of \(PP'\) is derived from (4) set equal to zero. Since \(U_T''(,) < 0\) and \(H > L\), it follows that \(0 < dL/dH = U_T'(H)/U_T'(L) < 1\). Decreasing absolute risk aversion implies that \(d^2L/dH^2 > 0\), so \(PP'\) is convex. As for the \(Bs\), we normalise so their utility of income equals income, and assume that at the endowment point, precautions increase expected wealth and hence utility by less than \(\overline{F}\). Hence, \(Bs\) never take precautions.

Indifference curves are drawn in income space assuming the optimal level of precautions is chosen. It follows that the indifference curves of the \(Ts\), labelled \(I_T\), are kinked where they cross \(PP'\). Above \(PP'\) the loss probability is raised so the indifference curves flatten. \(EE'\) is an indifference curve of a \(B\), which is linear since the \(Bs\) are risk neutral and \(\overline{F}\) is sufficiently high that in the relevant range they never take precautions.

The location of the insurers' zero-profit offer curves depends on the level of administrative costs, \(C\). When all applicants take precautions, the zero-profit offer curve is \(JJ'\), and \(JM'\) is the full-pooling offer curve given that the \(Ts\) take precautions and the \(Bs\) do not. Finally, \(JN'\) is the offer curve when no applicant takes precautions. The reason that \(J\) lies below \(E\) is the need to cover processing costs.

In identifying equilibria, it is evident that, when the administrative cost is sufficiently high, insurance is not viable and all agents remain at their endowment point. We show that at lower levels of \(C\), a separating equilibrium exists. As \(C\)
falls further, partial-pooling emerges as an equilibrium.

The propositions which follow display how the nature of equilibrium depends on \( C \). To conserve space and a sense of proportion, not every case is dealt with. In all that follows we assume that even though the \( T \)'s take steps to limit their risk exposure, at \( E \) their indifference curve is flatter than that of a \( B \) not taking precautions:

\[
(A1) \quad \left[ \frac{U_T'(H)}{U_T'(L)} \right] \left[ \frac{p_F}{(1 - p_F)} \right] < \left[ \frac{p_0}{(1 - p_0)} \right]
\]

Assumption A1 is required for a partial-pooling equilibrium.

The features of the equilibria in our model also depend upon the location of the tangency between \( EE' \) and the indifference curve of a \( T \) taking precautions. Note that, assuming an Inada condition, then a tangency must exist. On the 45° line both types of client are locally risk neutral, so if precautions were taken, the indifference curve of a \( T \) is steeper than \( EE' \), reflecting the higher loss probability of a \( B \). As \( L \) approaches zero, \( T \)'s indifference curve tends to flatness as the marginal utility of income in that state tends to infinity. Whether the tangency occurs between \( PP' \) and \( E \) depends on the magnitude of \( F \) and \( (H, L) \). Here we only consider in detail cases where the tangency of \( EE' \) and \( B \)'s indifference curve (denoted \( X \)) lies to the right of \( PP' \). The results when the tangency lies to the left of \( PP' \) are briefly summarised later.\(^8\)

We start with the configuration yielding a partial-pooling equilibrium in which those who take precautions subsidise the entry of those who do not.\(^9\) The numbers of \( T \)'s and \( B \)'s buying insurance policies is thus determined endogenously and

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\(^8\)The full analysis of this case is available from the authors.

\(^9\)Dionne and Doherty (1994) cast doubt on the extent of cross-subsidisation in insurance markets by observing the multiplicity of premia offers in the Californian automobile insurance market; Puelz and Snow (1994) obtain similar results for Georgia. However, if there is double crossing of indifference curves, as here, existence of even a continuum of offers may involve the same choice made by different types.
depends upon the cross-subsidy implied by the insurance contract relative to the expected administrative cost of insurance.

As the model is presently specified, there is potentially a continuum of positive-profit partial-pooling equilibria (see also Wambach (1997)). This is an artifact of the discreteness of the model and occurs if there are pooling offers at which the Bs are indifferent between purchase and remaining at E. Suppose that all companies offer a contract which is zero profit when taken by all Ts and Bs. The Bs are indifferent as to whether they purchase and suppose only some of the Bs choose to do so. Positive profits would then be earned by the contract. Nevertheless, no insurer would undercut by an epsilon, for the consequence is that all the Bs then strictly prefer to purchase and their high claim rate eliminates profits. This knife-edge feature reflects an extreme but inessential modelling assumption and is not of central economic interest. To ensure that only zero-profit equilibria emerge, the model is modified to introduce some vanishingly small heterogeneity between agents of each type:

(A2) Each agent $i$ has a utility cost $\varepsilon_i$ in applying for a policy. The distribution of $\varepsilon_i$ in the population is continuous with support $[0, \bar{\varepsilon}]$, where $\bar{\varepsilon}$ is arbitrarily small.

The $\varepsilon$ could be thought of as the effort cost of filling in a proposal form. The role of the $\varepsilon$s is to remove the discontinuity in the best-response functions and so eliminate positive-profit equilibria.\(^\text{10}\)

(Figure 1. here)

**Proposition 1.** If $C$ is sufficiently low that $JJ'$ cuts $EE'$ to the right of $X$ but $JM'$ does not cut $I_{Tj}^*$ to the right of $PP'$, and $N'$ lies below $I_{Tj}^*$, there exists a unique

\(^{10}\text{In all the equilibria we examine, the Ts are strictly better off if they purchase insurance so the } \varepsilon \text{s have no effect on their decisions.}
equilibrium. The equilibrium insurance contract, \( z_p \), is partial-pooling and located above but arbitrarily close to \( X \). More precisely, the contract maximises the utility of the \( T \)s subject to the insurers attracting the number of \( B \)s required for breakeven. It is therefore located at the tangency between a \( T \) and \( B \) indifference curve, just above \( X \).

**Proof.** It is trivial that \( z_p \) is an equilibrium. All \( T \)s are strictly better off if they take \( z_p \) rather than going uninsured, whilst the fraction of \( B \)s preferring \( z_p \) to \( E \) is just enough to render \( z_p \) zero profit granted that only \( T \)s take precautions. Now suppose an insurer deviated by offering a contract below \( PP' \). Due to the tangency, to attract any customers, an offer must be more attractive than \( z_p \) to \( B \)s, but if only \( B \)s take the proposed contract, it is certainly loss making. The only offers which are also taken by \( T \)s must involve losses since, by construction, \( z_p \) maximises the \( T \)'s utility subject to breakeven. If a deviation lies above \( PP' \), the most profitable offer must involve full insurance, but even the breakeven offer at \( N' \) is inferior to \( z_p \) for the \( T \)s. Thus there is no profitable deviation. For uniqueness, note that by construction, \( T \)s prefer \( z_p \) to any other zero-profit offer in which at least some \( T \)s participate, so it is the only possible pooling offer. Moreover, a separating offer to the \( T \)s must be on \( JJ' \) and on or below \( EE' \) and so would be destroyed by \( z_p \). Uniqueness follows. *Q.E.D.*

In the interval defined in Proposition 1, the insurance contract is effectively invariant to the level of \( C \), but the lower is \( C \), the more \( B \)s are insured.

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11 The contract actually at \( X \) is not an equilibrium. No \( B \)s purchase due to the application cost but as \( X \) lies below \( JJ' \), it is strictly profitable. An insurer would therefore gain by making a slightly more generous offer that captures all the \( T \)s, even though a few extra \( B \)s also purchase.

12 Eliminating the undercutting incentive must involve a zero-profit offer and so involve some but not all \( B \)s participating (since \( JM' \) lies below \( X \)). As the distribution of \( \varepsilon \) is compressed, this equilibrium is arbitrarily close to \( X \). Just enough \( B \)s buy the contract to render zero expected profit for the insurance company (so the contract satisfies 7(a)) and the tangency occurs an epsilon above \( X \).
If $C$ is above this interval, then, as shown in Figure 2, $JJ'$ cuts $EE'$ at or below $PP'$ but to the left of the tangency of $I_T^*$ and $EE'$. At this intersection, the indifference curve of the $Ts$ is flatter than $JJ'$ and a separating equilibrium arises.

(Figure 2. here)

Proposition 2. Suppose $C$ is sufficiently low that $JJ'$ cuts $EE'$ at or below $PP'$, but is high enough that $JJ'$ does not cut $EE'$ to the right of $X$, and $I_T'$, the $Ts$ indifference curve through the intersection point of $JJ'$ and $EE'$, does not cut $JM'$ to the right of $PP'$ and lies above $N'$. Then there exists a unique separating equilibrium with the $Ts$ contract, $z_T$, at or arbitrarily close to the intersection of $JJ'$ and $EE'$. The $Bs$ are uninsured.

Proof. At $z_T$ no $Bs$ purchase insurance due to the application cost. Now test $z_T$ by considering deviations. Offers above $JJ'$ are unprofitable even if no $Bs$ buy. No profitable offer below $PP'$ and below $I_T'$ attracts the $Ts$. There is, however, a zone bounded by $I_T'$ and $JJ'$ in which offers attract all $Ts$ and some $Bs$. The issue is whether so many $Bs$ are attracted that such offers are unprofitable. If the distribution of the $es$ is sufficiently compressed, any break-even offer that involves lower good-state income than at $z_T$ must be arbitrarily close to $EE'$ and therefore below $I_T'$. Hence, the pair of contracts $z_T$ and the endowment point is a separating equilibrium. For uniqueness, note that $z_T$ is preferred by the $Ts$ to any other offer on or below $JJ'$ and $EE'$. So there can be no other separating equilibrium. Full-pooling is ruled out since $I_T'$ does not cut $JM'$ to the right of $PP'$. Partial-pooling offers must be between $z_T$ and $PP'$ close to $EE'$ but, with a compressed $e$ distribution, would be broken by $z_T$. Uniqueness follows. Q.E.D.

\footnote{If the distribution of $es$ is insufficiently compact there will be a partial-pooling equilibrium above but close to $z_p$.}
Notice that separation is achieved with the Ts under-insured relative to the equilibrium with full information about types. Also, this zone has the counter-intuitive property that the lower is $C$, the less insurance coverage taken by Ts.\footnote{Note that given risk aversion the indifference curve of the Ts' is tangent to $JJ'$ when income is the same in both states, ruling out interior separating equilibria.}

(Figure 3, here)

Now suppose that $C$ exceeds the interval identified in Proposition 2 but is still not prohibitive. As illustrated in Figure 3, equilibrium now lies at the intersection of $JJ'$ and $PP'$ provided this puts the Ts on an indifference curve which passes above $N'$. There is then a unique equilibrium. It is separating with all the Ts taking the insurance contract at the intersection and the Bs uninsured. If the indifference curve passes below $N'$, the Ts take the without-precautions full-insurance contract at $N'$.

When administrative costs are low it becomes an issue whether a pure-strategy sub-game perfect Nash equilibrium exists. Consider first the configuration of Figure 1 where Proposition 1 establishes that a partial pooling equilibrium exists at $X$. Now let $C$ fall so that $J$ slides up $E\overline{H}$. There is some threshold value of $C$ below which $JM'$ still passes below $X$ but cuts $I_T^V$ below $PP'$ (when $JM'$ passes through $X$ it certainly cuts $I_T^V$ ). In this zone, there is no equilibrium. There will be some offers along $JM'$ that are better for Ts than $X$, and so break the partial pooling offer (which in turn breaks full pooling at the intersection of $JJ'$ and $EE'$). Moreover, full pooling on $JM'$ above $EE'$ is also ruled out as an equilibrium. Since the indifference curve of a $B$ is flatter than that of a $T$ at such points, a small deviation to less coverage attracts only $T$s, so is certainly profitable. This argument applies even when there is no tangency between $JM'$ and the indifference curve of a $T$. The only candidate equilibrium is then where
$JM'$ and $PP'$ intersect but since the $Bs$ indifference curve is flatter than the $Ts$, there must exist deviations to lower coverage that yield profitable separation.

When $C$ is so low that $JM'$ passes above $X$ a similar argument applies. Separation is again broken by offers along $JM'$ and above $EE'$. Full pooling on $JM'$ is ruled out as an equilibrium. There is now a zone along $JM'$ where the indifference curves of the $Ts$ are flatter than those of the $Bs$, so here pooling is broken by deviating to an offer involving a little more coverage, which profitably, only attracts $Ts$.  

Even when $C$ is sufficiently high that $X$ lies below $JJ'$, non existence may arise. Figure 2 can be modified so that $JM'$ cuts $I_T$ below $PP'$. Now there are offers along $JM'$ that $Ts$ prefer to the separating contract at $z_T$, which is therefore eliminated as an equilibrium. However, as previously, offers on $JM'$ can in turn be broken by deviations that only attract $Ts$.

To summarise, the comparative statics of our model as the administration cost, $C$, changes are as follows. At very high values of $C$ no insurance is purchased. As $C$ falls, a zone is entered in which separating equilibria exist. Here, the $Bs$ do not purchase insurance and the $Ts$ take either full or partial insurance. As $C$ falls further, partial-pooling emerges and finally, for $C$ sufficiently low, there exists no sub-game perfect Nash equilibrium. 

Finally, we sketch outcomes when there is no tangency between the $Ts'$ indifference curve and $EE'$ to the right of $PP'$. All these “single-crossing” cases involve equilibrium on $PP'$ (assuming such contracts are not dominated for the

\footnote{There is a point on $JM'$ above $EE'$ where the indifference curves of $Bs$ and $Ts$ are tangent, but then insurance companies can make profitable deviations to offers below $JM'$ that make even $Ts$ better off.}

\footnote{Existence can always be restored if the equilibrium concept is Wilson rather than Nash (see Wilson (1977)). This is not a novel observation, the interest being that, using arguments along the lines of those in Section 3, such equilibria can be shown to exhibit excessive coverage.}
Ts by the without-precautions full-insurance offer at $N'$. When administrative costs are high but not prohibitive, so that $JJ'$ cuts $PP'$ below $EE'$, there exists a separating equilibrium in which the $Ts$ are partially insured. The insurance contract lies at the intersection of $JJ'$ and $PP'$ and the $Bs$ go uninsured. At lower values of $C$, $JJ'$ cuts $PP'$ above $EE'$, but $JM'$ cuts $PP'$ below $EE'$. There then exists a partial-pooling equilibrium at the intersection of $PP'$ and $EE'$.

At still lower values of $C$, $JM'$ cuts $PP'$ above $EE'$, at which intersection there is a full-pooling equilibrium.\(^{17}\)

Looking to the most interesting cases, the existence of zero-profit partial or full pooling equilibria depends upon the marginal buyer being the boldest and highest risk of those active. In our formulation, zero profit can be achieved with incomplete participation by the marginal types. What allows pooling is the double crossing of indifference curves, which becomes possible once precautions are endogenous. In a pooling configuration, a small cut in the premium or alteration in coverage causes a flood of bold entrants, taking no precautions, and therefore is unprofitable. The conventional model sees a cut in the premium leading to an influx of good risks and it thus yields an increase in profit, thereby precluding a partial-pooling equilibrium.

Just as in the conventional model, separating equilibria may also arise. This occurs when there is single crossing, or, as in Proposition 2, if double crossing does not apply in the relevant zone. The difference with the standard model is that here the good risks make their contracts undesirable to the bad risks by raising coverage from the full information level.\(^{18}\)

\(^{17}\)This is an artifact of the discreteness of precautionary choice, but even when precautions are continuous, single crossing permits equilibria in which bold types are overinsured relative to the full information equilibrium.

\(^{18}\)Our main results apply if the marginal buyer is risk neutral and if precautionary effort is a continuous variable.
3. Welfare

In our set up, it is possible to find policies that yield strict Pareto gains. Consider the equilibrium of Proposition 1 in which entry of the Bs has taken place up to the point at which they gain no surplus from insurance and some, but not all, are uninsured.

(Figure 4. here)

Proposition 3. In the partial-pooling equilibrium of Proposition 1, introducing a small fixed tax per policy issued, with the proceeds returned as a lump-sum subsidy to the whole population, yields a strict Pareto improvement.

Proof. The subsidy shifts the endowment point up the 45° line to $\hat{E}$ and the new indifference curve of a type B is $\hat{E}\hat{E}'$. The tax raises the fixed cost of insurance to $\hat{C}$. As the subsidy is received by all, but the tax is only paid by those buying insurance, endowments rise by less than the cost of insurance, so $J$ moves down the 45° line to and the new zero profit curve is $\hat{J}\hat{P}'$. The new equilibrium is at $z'_p$, arbitrarily close to $\hat{X}$, which is preferred by all. Q.E.D.

Note that the premium per dollar of coverage remains the same despite the tax. This is possible since fewer higher risk Bs are insured. Also, for a sufficiently large tax, the pooling equilibrium breaks down and separation results.

If the laissez-faire equilibrium is separating, as in Proposition 2, the possibility of a strict Pareto improvement again arises. The tax makes it less attractive for the Bs to purchase insurance and so allows the Ts to achieve separation with increased insurance coverage. More specifically, the tax on each policy and return of the proceeds as a poll subsidy slides the zero coverage contract, $J$, down the 45° line to $\hat{J}$ and the subsidy to the uninsured shifts the endowment point $E$ up
to \( \hat{E} \) (definitely benefiting Bs). The contract at the intersection of \( \hat{J}\hat{J}' \) and \( \hat{E}\hat{E}' \), the new equilibrium, lies to the North West of the initial contract. The question is whether the Ts have gained?

(Figure 5, here)

*Proposition 4.* In the separating equilibrium of Proposition 2, introducing a fixed tax per policy issued, \( \tau \), with the proceeds returned as a lump-sum subsidy of \( s \) to the whole population yields a strict Pareto improvement if the absolute value of the slope of the Ts’ indifference curve at \( E \) is less than \( \frac{p_0 + (p_F - p_0)(1 - \tau)}{(1 - p_0) - (p_F - p_0)(1 - \tau)} \), where \( r \) is the fraction of the population that are Bs.

*Proof.* The balanced budget condition is

\[ s = \tau (1 - r) \tag{8} \]

The equation of \( \hat{E}\hat{E}' \) is

\[ p_0(W - y + s) + (1 - p_0)(W - D + \lambda y + s) = p_0(W + s) + (1 - p_0)(W - D + s) \tag{9} \]

which implies that

\[ \lambda = \frac{p_0}{1 - p_0} \tag{10} \]

The equation of \( \hat{J}\hat{J}' \) is

\[ p_F - (1 - p_F)(\lambda y + C) = \tau \tag{11} \]

Substituting (10) into (11) yields

\[ y = \frac{(\tau + (1 - p_F)C)}{p_F - p_0} (1 - p_0), \lambda y = \frac{(\tau + (1 - p_F)C)}{p_F - p_0} p_0 \tag{12} \]
Now
\[ EU_T = p_F U (W - y + s) + (1 - p_F) U (W - D + \lambda y + s) \]  \hspace{1cm} (13)

Using (8) and (12), \( \frac{dE_U}{dT} > 0 \) if
\[ \frac{p_F U'(H)}{(1 - p_F) U'(L)} < \frac{p_0 + (p_F - p_0) (1 - r)}{(1 - p_0) - (p_F - p_0) (1 - r)} \] \hspace{1cm} (14)

The left hand side of (14) is the absolute value of the slope of \( T_s \) indifference curve at \( E \). The slope of the locus of intersections generated by balanced-budget variations of the tax is given by the term on the right hand side of (14) and varies between the slope of \( \tilde{J}' \) and \( EE' \) as \( r \) ranges from zero to one. So, when there are sufficiently few \( B_s \) in the population, a small tax makes everyone strictly better off. Q.E.D.

The reason the tax is more effective when there are few \( B_s \) is that the per capita subsidy is then high and so there is a large effect on the utility of the uninsured \( B_s \) and hence also on the \( T_s \).

4. Conclusions

Unlike the standard insurance model, the formulation developed here may yield a unique, sub-game perfect, partial-pooling, Nash equilibrium with the property that insurance market failure may be in the direction of excessive provision. Separating equilibria with this property are also possible. The key to these results is that, as premiums rise, it is the least risk-averse types who drop out of the market, the very people most inclined to reckless behaviour. Thus, the marginal purchasers impose an externality on the other buyers and it would be better if they were not in the market. Indeed, there exist feasible schemes which make
everyone better off. The argument was made explicit using the simplest model possible, but is clearly more general.

Casual observation does suggest that the worst risks often do without insurance, whereas in the standard model it is the good risks that are not covered. The evidence cited in the Introduction that the insured are less likely to suffer losses is consistent with this view.\footnote{Chiappori (1998) includes a useful overview of the evidence.} Chiappori and Salanie (2000) find though that the accident rate of drivers choosing comprehensive insurance is not significantly different from those opting for the legal minimum of third-party coverage. Although our model implies that the comprehensively insured have the lowest accident rates, minor modification allows for equality. Comprehensive insurance allows claims to be made for contingencies not covered by a third party policy, and so entails higher expected administrative costs which, in our analysis, can be represented by $C$. In a partial-pooling equilibrium, some bold types purchase comprehensive policies and others opt for the third-party coverage. Due to moral hazard, selecting a comprehensive policy induces fewer precautions than if third party coverage is chosen.\footnote{This assumes at least three levels of precautionary effort.} So, comprehensive insurance is taken by the safest drivers of all, the timid, and also by those with the very worst accident rates, bold types with no incentive to take care. Average accident rates may thus be the same for holders of the two policies. Whereas the Rothschild and Stiglitz model of adverse selection is inconsistent with Chiappori and Salanie’s findings, the more so if moral hazard is added, advantageous selection plus moral hazard does potentially account for them.

Cawley and Philipson (1999) also report the striking observation that insurance premiums display quantity discounts, the opposite of the prediction of the standard model in which those buying the most insurance are the bad risks. In
our model, the bold types are risk neutral so, in the absence of intervention, in a separating equilibrium they do not buy insurance at all. If, instead, we assumed that the bold, though more risk tolerant than the timid types, were nevertheless risk averse, separation may involve both types buying some insurance. The bold types purchase less coverage and take fewer precautions, so breakeven premiums on their policies must involve a higher loading factor. Even without fixed administrative costs, a quantity discount emerges.

The key to our resolution of the puzzling empirical features of insurance markets is that people who are reluctant to purchase insurance are also reluctant to take precautions. We argued that heterogeneous risk preference with endogenous precautionary effort could lead to just such a correlation. Other explanations are possible. There is a wealth of psychological evidence that people tend to be unrealistically optimistic concerning the probability of suffering losses, particularly when events are perceived as under the individual’s control. Indeed, Adam Smith regarded the failure of people to take out insurance as the result of “thoughtlessness rashness and presumptuous contempt of risk” (Wealth of Nations Book 1, Ch. X). It is easily seen that such unrealistic optimists will be less inclined to purchase insurance and even if they do so they may well take fewer precautions. The positive results of the paper hold for this case, but of course

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21 Similar results apply even if there is no causal link between risk attitudes and loss propensity. For example, it could be that there is no moral hazard but clumsy people happen to be the least risk averse. There is no obvious reason why this should be so, nor even stylized evidence in favour.


23 Rutter, Quine and Alberry (1998) find that motorcyclists are generally prone to overoptimism concerning the chance of accident but there is no clear tendency for those taking the most safety precautions to perceive lower absolute risks. This suggests that the most reckless riders are optimists.
the welfare results are reversed; now the equilibrium involves too little insurance. Consider the implications if unrealistic optimism is heterogeneously distributed. The most optimistic types will tend to be the least willing to purchase insurance. It is also possible that an attitude of ‘it won’t happen to me’ is a discouragement to take precautions. A misperception that risks are already low means a belief that few precautions are necessary is an attitude which may be reinforced by overestimation of the efficacy of what actions are taken. The net result is similar to heterogeneous risk preferences; the marginal insurance buyers are the riskiest of all and separating and pooling equilibria paralleling those analysed here may emerge. The major difference concerns policy. Optimism implies a mistaken reluctance to purchase insurance. The cross subsidy which draws in marginal types may now be insufficient to offset the cognitive bias that leads to underinsurance where there are no hidden types. The policy analysis is by no means straightforward and we do not pursue it here.24

Finally, although our discussion has been in terms of insurance markets, other agency problems have similar features. The design of managerial compensation schemes, corporate finance issues, and selection into self employment may be fruitful applications of the approach.

References


24There is evidence that people overweight the probability of rare events (e.g. Kahneman and Tversky, (1979)) in which case insurance purchase against such eventualities would be subject to unrealistic pessimism. As long as this trait is unequally distributed, a version of our positive analysis applies.


Fig. 2 of 5
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